$$J = D + c \cdot \underset{\sim}{1}\,\underset{\sim}{1}^T$$

where, $D = \operatorname{diag}\left\{ 1/\alpha_1, \dots, 1/\alpha_{K-1} \right\}$

$$c = \frac{1}{1 - \sum_{j=1}^{K-1} \alpha_j}$$

Goal is to find J^{-1}

[General formula : $(A + \underset{\sim}{u}\,\underset{\sim}{v}^T)^{-1}$

$$= A^{-1} - \frac{A^{-1}\,\underset{\sim}{u}\,\underset{\sim}{v}^T A^{-1}}{(1 + \underset{\sim}{v}^T A^{-1} \underset{\sim}{u})}$$]

$$J^{-1} = D^{-1} - \frac{D^{-1}\, c\, \underset{\sim}{1}\,\underset{\sim}{1}^T D^{-1}}{(1 + c\,\underset{\sim}{1}^T D^{-1} \underset{\sim}{1})}$$

[$\underset{\sim}{u} = c\,\underset{\sim}{1}$
$\underset{\sim}{v} = \underset{\sim}{1}$]

$$= \begin{pmatrix} \alpha_1 & & \\ & \ddots & \\ & & \alpha_{K-1} \end{pmatrix} - e \cdot \begin{pmatrix} \alpha_1 \\ \vdots \\ \alpha_{K-1} \end{pmatrix} (\alpha_1 \dots \alpha_{K-1})$$

[$\underset{\sim}{1}^T D^{-1} \underset{\sim}{1}$
$= \underset{\sim}{1}^T \begin{pmatrix} \alpha_1 & & \\ & \ddots & \\ & & \alpha_{K-1} \end{pmatrix}^{-1} \underset{\sim}{1}$
$= \sum_{i=1}^{K-1} \alpha_i$]

$$e = \frac{c}{1 + c \sum_{i=1}^{K-1} \alpha_i}$$

$$= \frac{\dfrac{1}{1 - \sum_{i=1}^{K-1} \alpha_i}}{1 + \dfrac{\sum_{i=1}^{K-1} \alpha_i}{1 - \sum_{i=1}^{K-1} \alpha_i}} = 1$$

$$J^{-1} = \begin{pmatrix} \alpha_1 & & \\ & \ddots & \\ & & \alpha_{K-1} \end{pmatrix} - \begin{pmatrix} \alpha_1 \\ \vdots \\ \alpha_{K-1} \end{pmatrix} (\alpha_1 \dots \alpha_{K-1})$$