THE DELTA-SIGMA TOOLBOX Version 7.3

Getting Started

The Delta-Sigma toolbox requires the `Signal Processing Toolbox` and the `Control Systems Toolbox`. Certain functions (`clans` and `designLCBP`) also require the `Optimization Toolbox`.

To obtain a copy of the Delta-Sigma toolbox, go to the MathWorks web site (`http://www.mathworks.com/matlabcentral/fileexchange`) and search for `delsig`. To improve simulation speed, compile the `simulateDSM.c` file by typing `mex simulateDSM.c` at the Matlab prompt. Do the same for `simulateESL.c`, `ai2mif.c`. (for importing figures into FrameMaker) and `flattenStruct.c`. (used by `dsexample4`).

For more detailed descriptions of selected functions and more examples, see Chapters 8 and 9 of

[1] R. Schreier and G. C. Temes, *Understanding Delta-Sigma Data Converters*, John Wiley & Sons, New York, 2004.

Toolbox Conventions

Frequencies are normalized; $f = 1$ corresponds to f_s.

Default values for function arguments are shown following an equals sign (=) in the parameter list. To use the default value for an argument, omit the argument if it is at the end of the list, otherwise use `NaN` (not-a-number) or `[]` (the empty matrix) as a place-holder.

A matrix is used to describe the loop filter of a general single-quantizer delta-sigma modulator. See “MODULATOR MODEL DETAILS” on page 33 for a description of this “ABCD” matrix.

Demonstrations and Examples

`dsdemo1`	Demonstration of the `synthesizeNTF` function. Noise transfer function synthesis for a 5^{th}-order lowpass modulator, both with and without optimized zeros, plus an 8^{th}-order bandpass modulator with optimized zeros.
`dsdemo2`	Demonstration of the `simulateDSM`, `predictSNR` and `simulateSNR` functions: time-domain simulation, SNR prediction using the describing function method of Ardalan and Paulos, spectral analysis and signal-to-noise ratio for lowpass, bandpass, and multi-bit lowpass examples.
`dsdemo3`	Demonstration of the `realizeNTF`, `stuffABCD`, `scaleABCD` and `mapABCD` functions: coefficient calculation and dynamic range scaling.
`dsdemo4`	Audio demonstration of MOD1 and MOD2 with sinc^n decimation.
`dsdemo5`	Demonstration of the `simulateESL` function: simulation of the element selection logic of a mismatch-shaping DAC.
`dsdemo6`	Demonstration of the `designHBF` function. Hardware-efficient halfband filter design and simulation.
`dsdemo7`	Demonstration of the `findPIS` function: positively-invariant set computation.
`dsdemo8`	Demonstration of the `designLCBP` function: continuous-time bandpass modulator design. (This function requires the `Optimization Toolbox`.)
`dsexample1`	Example discrete-time lowpass modulator.
`dsexample2`	Example discrete-time bandpass modulator.
`dsexample3`	Example continuous-time lowpass modulator.
`dsexample4`	Example discrete-time quadrature bandpass modulator.

KEY FUNCTIONS

`ntf = synthesizeNTF(order=3,R=64,opt=0,H_inf=1.5,f0=0)` page 5
`ntf = clans(order=4,R=64,Q=5,rmax=0.95,opt=0)` page 6
`ntf = synthesizeChebyshevNTF(order=3,R=64,opt=0,H_inf=1.5,f0=0)` page 7
Synthesize a noise transfer function.

`[v,xn,xmax,y] = simulateDSM(u,ABCD` or `ntf,nlev=2,x0=0)` page 8
Simulate a delta-sigma modulator with a given input.

`[snr,amp] = simulateSNR(ntf,R,amp=...,f0=0,nlev=2,f=1/(4*R),k=13)` page 9
Determine the SNR vs. input power curve by simulation.

`[a,g,b,c] = realizeNTF(ntf,form='CRFB',stf=1)` page 10
Convert a noise transfer function into coefficients for a specific structure.

`ABCD = stuffABCD(a,g,b,c,form='CRFB')` page 11
Calculate the ABCD matrix given the parameters of a specified modulator topology.

`[a,g,b,c] = mapABCD(ABCD,form='CRFB')` page 11
Calculate the parameters of a specified modulator topology given the ABCD matrix.

`[ABCDs, umax] = scaleABCD(ABCD,nlev=2,f=0,xlim=1,ymax=nlev+2))` page 12
Perform dynamic range scaling on a delta-sigma modulator described by ABCD.

`[ntf,stf] = calculateTF(ABCD,k=1)` page 13
Calculate the NTF and STF of a delta-sigma modulator described by the ABCD matrix, assuming a quantizer gain of *k*.

`[ABCDc,tdac2] = realizeNTF_ct( ntf, form='FB', tdac, ordering=[1:n], bp=zeros(…), ABCDc)` page 14
Realize an NTF with a continuous-time loop filter.

`[sys, Gp] = mapCtoD(sys_c,t=[0 1],f0=0)` page 15
Map a continuous-time system to a discrete-time system whose impulse response matches the sampled pulse response of the original continuous-time system. See `dsexample3`.

`H = evalTFP(Hs,Hz,f)` page 16
`evalTFP` computes the value of the transfer function product `Hs*Hz` at a frequency `f`. Use this function to evaluate the signal transfer function of a CT system.

`[sv,sx,sigma_se,max_sx,max_sy]=simulateESL(v,mtf,M=16,dw=[1…],sx0=[0…])` page 17
Simulate the element-selection logic in a mismatch-shaping DAC.

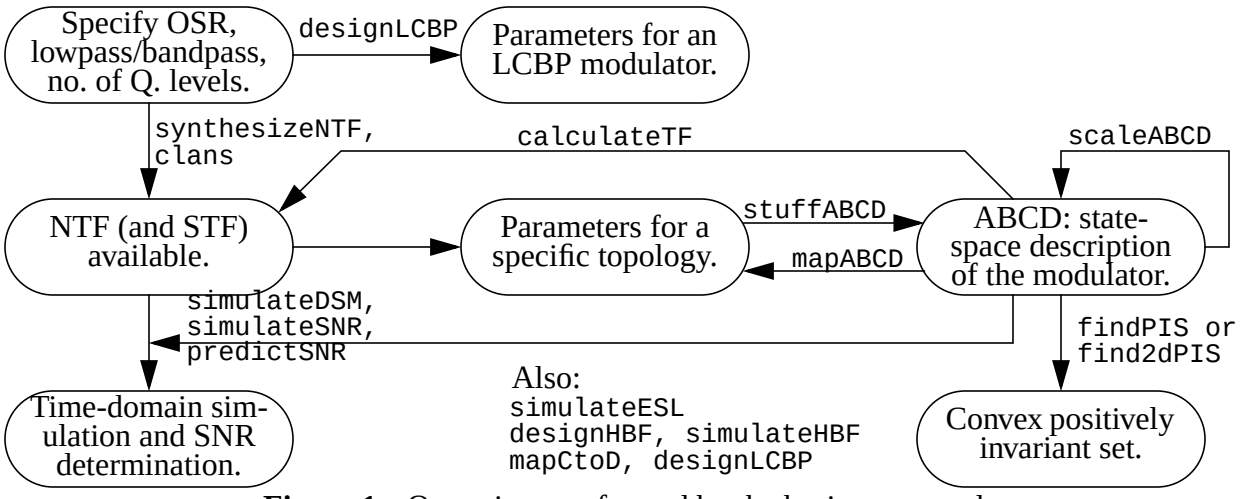


Figure 1: Operations performed by the basic commands.

FUNCTIONS FOR QUADRATURE SYSTEMS

Note: `simulateSNR` works for a quadrature modulator if given a complex NTF or ABCD matrix; `simulateDSM` can be used for a quadrature modulator if given an ABCDr matrix.

SPECIALTY FUNCTIONS

OTHER SELECTED FUNCTIONS

Delta-Sigma Utility

`mod1, mod2`
Set the `ABCD` matrix, `NTF` and `STF` of the standard 1st and 2nd-order modulators.

`snr = calculateSNR(hwfft,f)`
Estimate the SNR given the in-band bins of a Hann-windowed FFT and the location of the input signal.

`[A B C D] = partitionABCD(ABCD, m)`
Partition `ABCD` into `A`, `B`, `C`, `D` for an *m*-input state-space system.

`H_inf = infnorm(H)`
Compute the infinity norm (maximum absolute value) of a z-domain transfer function. See `evalTF`.

`y = impL1(ntf,n=10)`
Compute *n* points of the impulse response from the comparator output back to the comparator input for the given NTF.

`y = pulse(S,tp=[0 1],dt=1,tfinal=10,nosum=0)`
Compute the sampled pulse response of a continuous-time system.

`sigma_H = rmsGain(H,f1,f2)`
Compute the root mean-square gain of the discrete-time transfer function H in the frequency band (`f1`,`f2`).

General Utility

`dbv(), dbp(), undbv(), undbp(), dbm(), undbm()`
The dB equivalent of voltage/power quantities, and their inverse functions.

`window = ds_hann(N)`
A Hann window of length `N`. Unlike MATLAB's original `hanning` function, `ds_hann` does not smear tones which are located exactly in an FFT bin (i.e. tones having an integral number of cycles in the given block of data). MATLAB 6's `hanning(N,'periodic')` function and MATLAB 7's `hann(N,'periodic')` function do the same as `ds_hann(N)`.

Graphing

`plotPZ(H,color='b',markersize=5,list=0)`
Plot the poles and zeros of a transfer function.

`plotSpectrum(X,fin,fmt)`
Plot a smoothed spectrum

`figureMagic(xRange,dx,xLab, yRange,dy,yLab, size)`
Performs a number of formatting operations for the current figure, including axis limits, ticks and labelling.

`printmif(file,size,font,fig)`
Print a figure to an Adobe Illustrator file and then use `ai2mif` to convert it to FrameMaker MIF format. `ai2mif` is an improved version of the function of the same name originally written by Deron Jackson <djackson@mit.edu>.

`[f,p] = logsmooth(X,inBin,nbin)`
Smooth the FFT `X`, and convert it to dB. See also `bplogsmooth` and `bilogplot`.

synthesizeNTF

Synopsis: `ntf = synthesizeNTF(order=3,OSR=64,opt=0,H_inf=1.5,f0=0)`
Synthesize a noise transfer function (NTF) for a delta-sigma modulator.

Arguments

order	The order of the NTF. *order* must be even for bandpass modulators.
OSR	The oversampling ratio. *OSR* need only be specified when optimized NTF zeros are requested.
opt	A flag used to request optimized NTF zeros. `opt=0` puts all NTF zeros at band-center (DC for lowpass modulators). `opt=1` optimizes the NTF zeros. For even-order modulators, `opt=2` puts two zeros at band-center, but optimizes the rest.
H_inf	The maximum out-of-band gain of the NTF. Lee's rule states that *H_inf*<2 should yield a stable modulator with a binary quantizer. Reducing *H_inf* increases the likelihood of success, but reduces the magnitude of the attenuation provided by the NTF and thus the theoretical resolution of the modulator.
f0	The center frequency of the modulator. `f0≠0` yields a bandpass modulator; `f0=0.25` puts the center frequency at $f_s/4$.

Output

ntf	The modulator NTF, given as an LTI object in zero-pole form.

Bugs

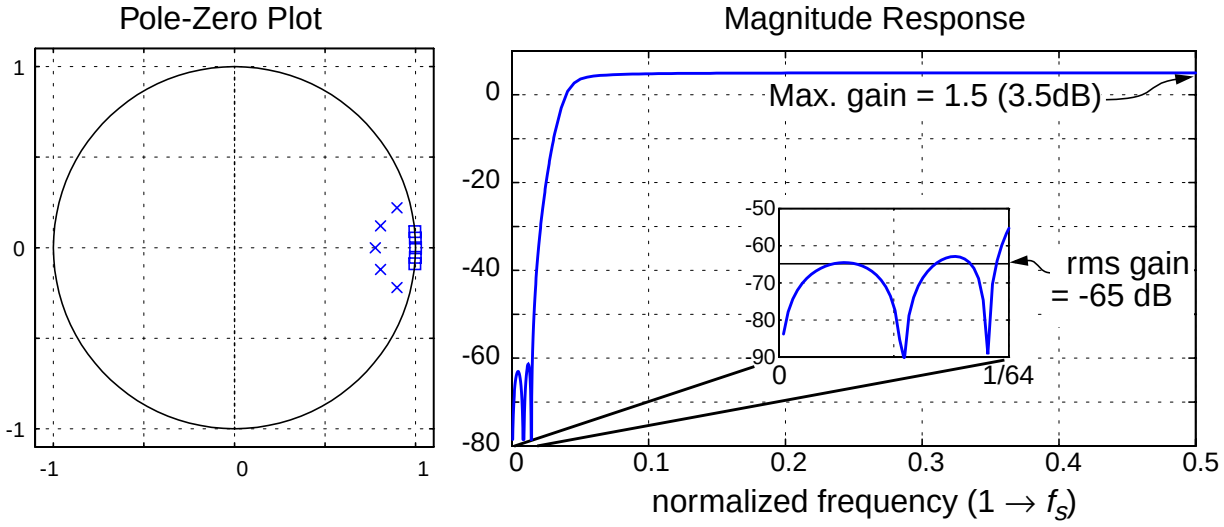
If *OSR* or *H_inf* are low, the NTF is not optimal. Use `synthesizeChebyshevNTF` instead.

Example

Fifth-order lowpass modulator; zeros optimized for an oversampling ratio of 32.

```
>> H = synthesizeNTF(5,32,1)
Zero/pole/gain:
       (z-1) (z^2 - 1.997z + 1) (z^2 - 1.992z + 1)
-----------------------------------------------------------
(z-0.7778) (z^2 - 1.613z + 0.6649) (z^2 - 1.796z + 0.8549)

Sampling time: 1
```



clans

Synopsis: `ntf = clans(order=4,OSR=64,Q=5,rmax=0.95,opt=0)`
Synthesize a noise transfer function (NTF) for a lowpass delta-sigma modulator using the CLANS (Closed-loop analysis of noise-shaper) methodology [1]. This function requires the optimization toolbox.

[1] J. G. Kenney and L. R. Carley, "Design of multibit noise-shaping data converters," *Analog Integrated Circuits Signal Processing Journal*, vol. 3, pp. 259-272, 1993.

Arguments

order	The order of the NTF.
OSR	The oversampling ratio.
Q	The maximum number of quantization levels used by the fed-back quantization noise. (Mathematically, $Q = \|h\|_1 - 1$, i.e. the sum of the absolute values of the impulse response samples minus 1, is the maximum instantaneous noise gain.)
rmax	The maximum radius for the NTF poles.
opt	A flag used to request optimized NTF zeros. `opt=0` puts all NTF zeros at band-center (DC for lowpass modulators). `opt=1` optimizes the NTF zeros. For even-order modulators, `opt=2` puts two zeros at band-center, but optimizes the rest.

Output

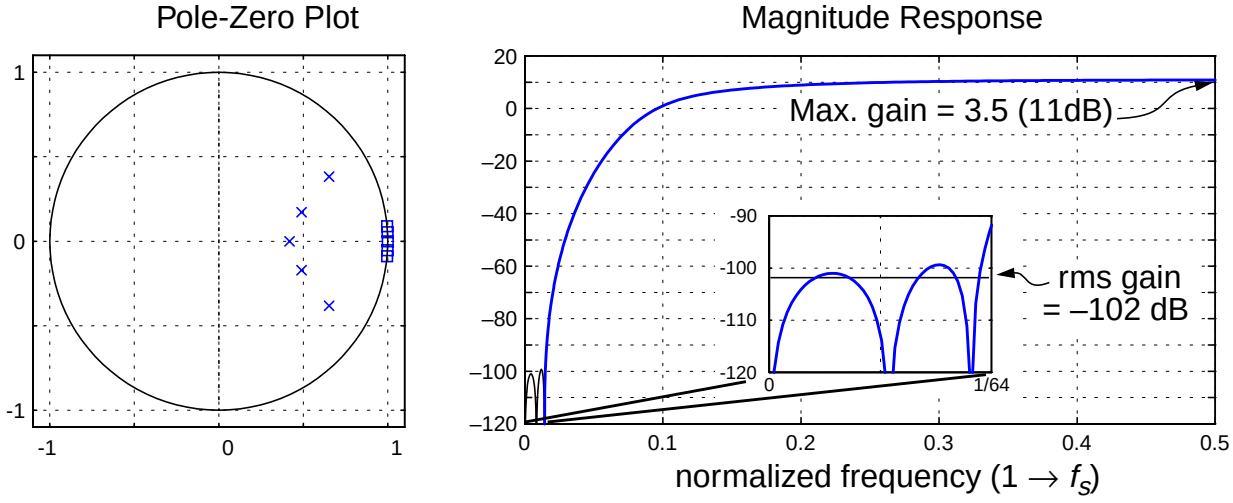
ntf	The modulator NTF, given as an LTI object in zero-pole form.

Example
Fifth-order lowpass modulator; (time-domain) noise gain of 5, zeros optimized for *OSR* = 32.

```
>> H= clans(5,32,5,.95,1)

Zero/pole/gain:
       (z-1) (z^2 - 1.997z + 1) (z^2 - 1.992z + 1)
-----------------------------------------------------------
(z-0.4183) (z^2 - 0.9784z + 0.2685) (z^2 - 1.305z + 0.5714)

Sampling time: 1
```



synthesizeChebyshevNTF

Synopsis: `ntf = synthesizeChebyshevNTF(order=3,OSR=64,opt=0,H_inf=1.5,f0=0)`
Synthesize a noise transfer function (NTF) for a delta-sigma modulator. `synthesizeNTF` assumes that magnitude of the denominator of the NTF is approximately constant in the passband. When the *OSR* or H_inf are low, this assumption breaks down and `synthesizeNTF` yields a non-optimal NTF. `synthesizeChebyshevNTF` creates non-optimal NTFs, but fares better than `synthesizeNTF` in the aforementioned circumstances.

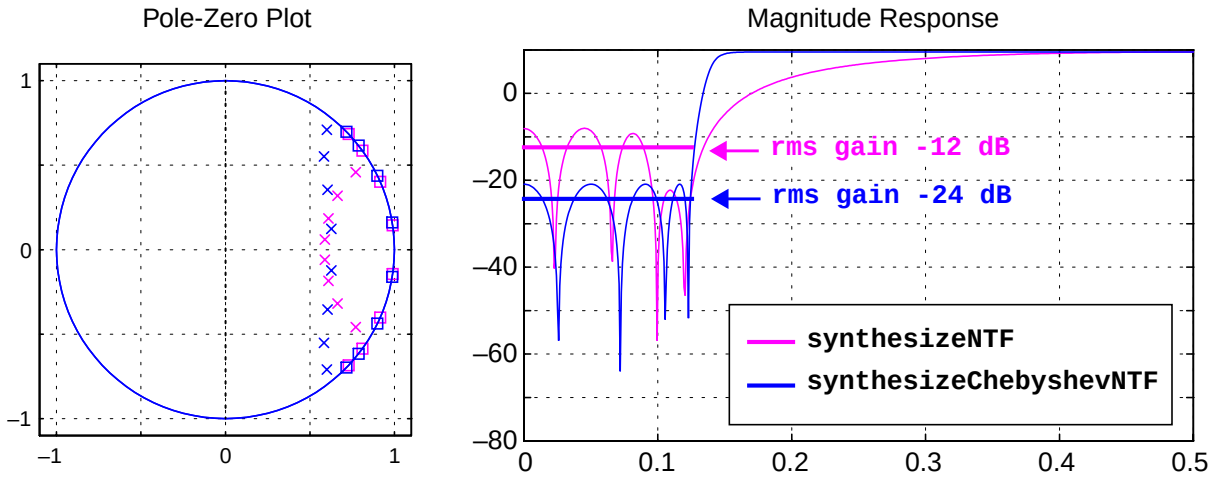
Arguments and Output

Same as `synthesizeNTF`, except that the *f0* argument is not supported yet.

Examples

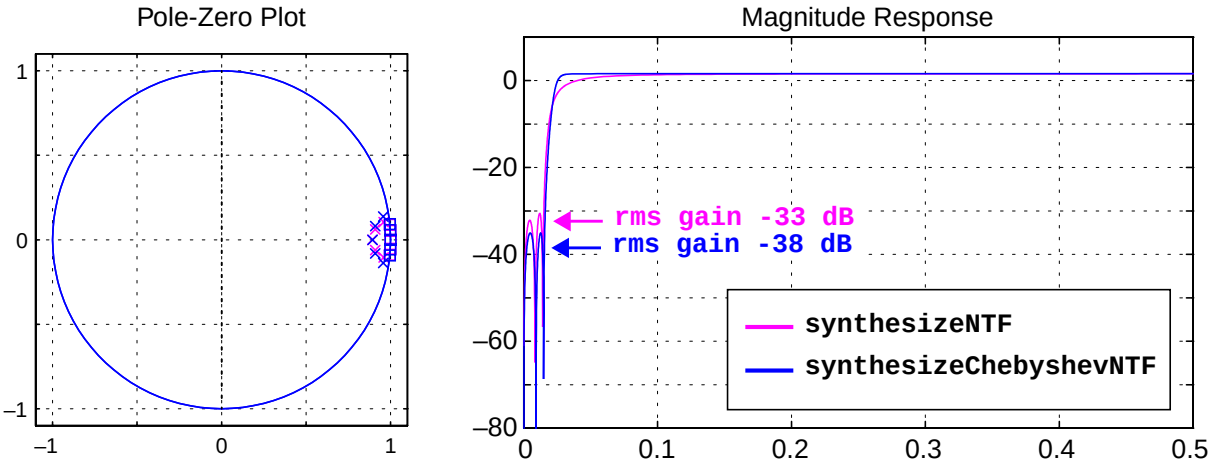
Compare the NTFs created by `synthesizeNTF` and `synthesizeChebyshevNTF` when *OSR* is low:

```
>> OSR = 4; order = 8; H_inf = 3;
>> H1 = synthesizeNTF(order,OSR,1,H_inf);
>> H3 = synthesizeChebyshevNTF(order,OSR,1,H_inf);
```



Repeat for *H_inf* low:

```
>> OSR = 32; order = 5; H_inf = 1.2;
>> H1 = synthesizeNTF(order,OSR,1,H_inf);
>> H3 = synthesizeChebyshevNTF(order,OSR,1,H_inf);
```



simulateDSM

Synopsis: `[v,xn,xmax,y] = simulateDSM(u,ABCD|ntf,nlev=2,x0=0)`
Simulate a delta-sigma modulator with a given input. For maximum speed, make sure that the compiled `mex` file is on your search path (by typing `which simulateDSM` at the MATLAB prompt).

Arguments

u	The input sequence to the modulator, given as a $m \times N$ row vector. *m* is the number of inputs (usually 1). Full-scale corresponds to an input of magnitude *nlev*–1.
ABCD	A state-space description of the modulator loop filter.
ntf	The modulator NTF, given in zero-pole form. The modulator STF is assumed to be unity.
nlev	The number of levels in the quantizer. Multiple quantizers are indicated by making *nlev* an array.
x0	The initial state of the modulator.

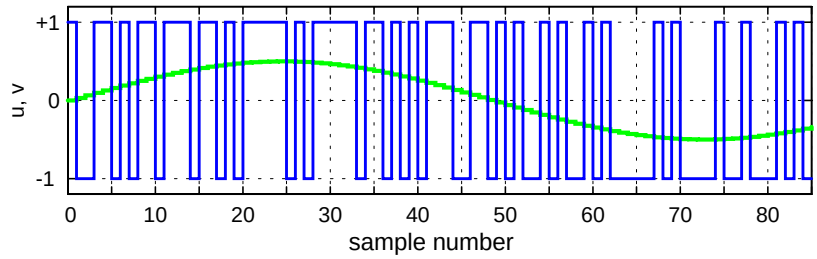
Output

v	The samples of the output of the modulator, one for each input sample.
xn	The internal states of the modulator, one for each input sample, given as an $n \times N$ matrix.
xmax	The maximum absolute values of each state variable.
y	The samples of the quantizer input, one per input sample.

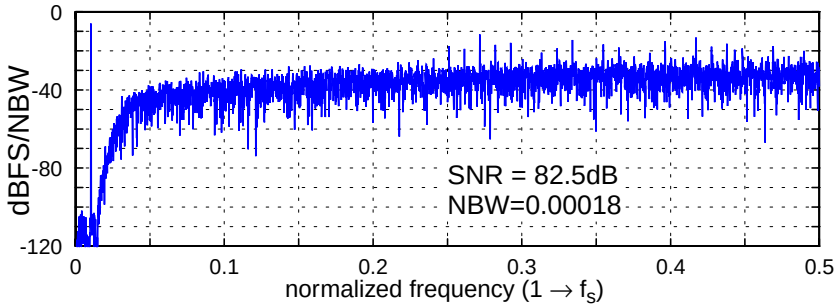
Example

Simulate a 5th-order binary modulator with a half-scale sine-wave input and plot its output in the time and frequency domains.

```
>> OSR = 32; H = synthesizeNTF(5,OSR,1);
>> N = 8192; fB = ceil(N/(2*OSR));
>> f=85; u = 0.5*sin(2*pi*f/N*[0:N-1]);
>> v = simulateDSM(u,H);
```




```
t = 0:85;
stairs(t, u(t+1),'g');
hold on;
stairs(t,v(t+1),'b');
axis([0 85 -1.2 1.2]);
ylabel('u, v');
```




```
spec=fft(v.*ds_hann(N))/(N/4);
plot(linspace(0,0.5,N/2+1), ...
  dbv(spec(1:N/2+1)));
axis([0 0.5 -120 0]);
grid on;
ylabel('dBFS/NBW')
snr=calculateSNR(spec(1:fB),f);
s=sprintf('SNR = %4.1fdB\n',snr)
text(0.25,-90,s);
s=sprintf('NBW=%7.5f',1.5/N);
text(0.25, -110, s);
```

simulateSNR

Synopsis: `[snr,amp] = simulateSNR(ntf|ABCD|function,OSR,amp,f0=0,nlev=2, f=1/(4*OSR),k=13,quadrature=0)`

Simulate a delta-sigma modulator with sine wave inputs of various amplitudes and calculate the signal-to-noise ratio (SNR) in dB for each input.

Arguments

ntf	The modulator NTF, given in zero-pole form.
ABCD	A state-space description of the modulator loop filter.
function	The name of a function taking the input signal as its sole argument.
OSR	The oversampling ratio. *OSR* is used to define the "band of interest."
amp	A row vector listing the amplitudes to use. Defaults to [-120 -110...-20 -15 -10 -9 -8 … 0] dB, where 0 dB means a full-scale (peak value = *nlev*-1) sine wave.
f0	The center frequency of the modulator. *f0*≠0 corresponds to a bandpass modulator.
nlev	The number of levels in the quantizer.
f	The normalized frequency of the test sinusoid; a check is made that the test frequency is in the band of interest. The frequency is adjusted so that it lies precisely in an FFT bin.
k	The number of time points used for the FFT is 2^k.
quadrature	A flag indicating that the system being simulated is quadrature. This flag is set automatically if either *ntf* or *ABCD* are complex.

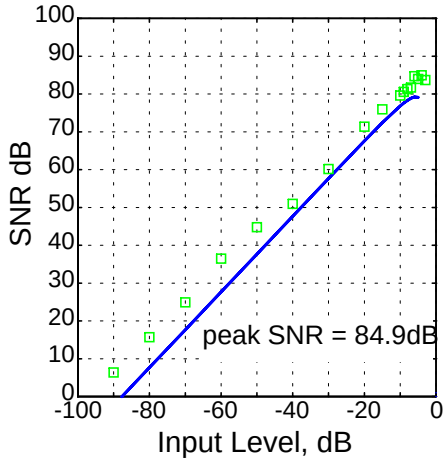
Output

snr	A row vector containing the SNR values calculated from the simulations.
amp	A row vector listing the amplitudes used.

Example

Compare the SNR vs. input amplitude curve for a fifth-order modulator determined by the describing function method with that determined by simulation.

```
>> OSR = 32; H = synthesizeNTF(5,OSR,1);
>> [snr_pred,amp] = predictSNR(H,OSR);
>> [snr,amp] = simulateSNR(H,OSR);
```




```
plot(amp,snr_pred,'b',amp,snr,'gs');
grid on;
figureMagic([-100 0], 10, 2, ...
   [0 100], 10, 1);
xlabel('Input Level, dB');
ylabel('SNR dB');
s=sprintf('peak SNR = %4.1fdB\n',...
   max(snr));
text(-65,15,s);
```

realizeNTF

Synopsis: `[a,g,b,c] = realizeNTF(ntf,form='CRFB',stf=1)`
Convert a noise transfer function (NTF) into a set of coefficients for a particular modulator topology.

Arguments

ntf The modulator NTF, given in zero-pole form (i.e. a zpk object).

form A string specifying the modulator topology.

CRFB	Cascade-of-resonators, feedback form.
CRFF	Cascade-of-resonators, feedforward form.
CIFB	Cascade-of-integrators, feedback form.
CIFF	Cascade-of-integrators, feedforward form.
… D	Any of the above, but the quantizer is delaying.

Structures are described in detail in “MODULATOR MODEL DETAILS” on page 33.

stf The modulator STF, specified as a zpk object. Note that the poles of the STF must match those of the NTF in order to guarantee that the STF can be realized without the addition of extra state variables.

Output

a Feedback/feedforward coefficients from/to the quantizer ($1 \times n$).

g Resonator coefficients ($1 \times \lfloor n/2 \rfloor$).

b Feed-in coefficients from the modulator input to each integrator ($1 \times n + 1$).

c Integrator inter-stage coefficients. ($1 \times n$. In unscaled modulators, *c* is all ones.)

Example

Determine the coefficients for a 5^{th}-order modulator with the cascade-of-resonators structure, feedback (CRFB) form.

```
>> H = synthesizeNTF(5,32,1);
>> [a,g,b,c] = realizeNTF(H,'CRFB')
a = 0.0007    0.0084    0.0550    0.2443    0.5579
g = 0.0028    0.0079
b = 0.0007    0.0084    0.0550    0.2443    0.5579    1.0000
c = 1     1     1     1     1
```

See Also

`realizeNTF_ct` on page 14 to realize an NTF with a continuous-time loop filter.

stuffABCD

Synopsis: `ABCD = stuffABCD(a,g,b,c,form='CRFB')`
Calculate the ABCD matrix given the parameters of a specified modulator topology.

Arguments

a	Feedback/feedforward coefficients from/to the quantizer. $1 \times n$
g	Resonator coefficients. $1 \times \lfloor n/2 \rfloor$
b	Feed-in coefficients from the modulator input to each integrator. $1 \times n + 1$
c	Integrator inter-stage coefficients. $1 \times n$
form	see `realizeNTF` on page 10 for a list of supported structures.

Output

ABCD	A state-space description of the modulator loop filter.

mapABCD

`[a,g,b,c] = mapABCD(ABCD,form='CRFB')`
Calculate the parameters for a specified modulator topology, assuming ABCD fits that topology.

Arguments

ABCD	A state-space description of the modulator loop filter.
form	see `realizeNTF` on page 10 for a list of supported structures.

Output

a	Feedback/feedforward coefficients from/to the quantizer. $1 \times n$
g	Resonator coefficients. $1 \times \lfloor n/2 \rfloor$
b	Feed-in coefficients from the modulator input to each integrator. $1 \times n + 1$
c	Integrator inter-stage coefficients. $1 \times n$

scaleABCD

Synopsis: `[ABCDs,umax]=scaleABCD(ABCD,nlev=2,f=0,xlim=1,ymax=nlev+5,umax,N=1e5)`
Scale the ABCD matrix so that the state maxima are less than a specified limit. The maximum stable input is determined as a side-effect of this process.

Arguments

ABCD	A state-space description of the modulator loop filter.
nlev	The number of levels in the quantizer.
f	The normalized frequency of the test sinusoid.
xlim	The limit on the states. May be given as a vector.
ymax	The threshold for judging modulator stability. If the quantizer input exceeds *ymax*, the modulator is considered to be unstable.

Output

*ABCD*s	The scaled state-space description of the modulator loop filter.
umax	The maximum stable input. Input sinusoids with amplitudes below this value should not cause the modulator states to exceed their specified limits.

calculateTF

Synopsis: `[ntf,stf] = calculateTF(ABCD,k=1)`
Calculate the NTF and STF of a delta-sigma modulator.

Arguments

ABCD	A state-space description of the modulator loop filter.
k	The value to use for the quantizer gain.

Output

ntf	The modulator NTF, given as an LTI system in zero-pole form.
stf	The modulator STF, given as an LTI system in zero-pole form.

Example

Realize a fifth-order modulator with the cascade-of-resonators structure, feedback form. Calculate the ABCD matrix of the loop filter and verify that the NTF and STF are correct.

```
>> H = synthesizeNTF(5,32,1)

Zero/pole/gain:
       (z-1) (z^2 - 1.997z + 1) (z^2 - 1.992z + 1)
-----------------------------------------------------------
(z-0.7778) (z^2 - 1.613z + 0.6649) (z^2 - 1.796z + 0.8549)

Sampling time: 1
>> [a,g,b,c] = realizeNTF(H)
a =
    0.0007    0.0084    0.0550    0.2443    0.5579
g =
    0.0028    0.0079
b =
    0.0007    0.0084    0.0550    0.2443    0.5579    1.0000
c =
     1     1     1     1     1
>> ABCD = stuffABCD(a,g,b,c)
ABCD =
    1.0000         0         0         0         0    0.0007   -0.0007
    1.0000    1.0000   -0.0028         0         0    0.0084   -0.0084
    1.0000    1.0000    0.9972         0         0    0.0633   -0.0633
         0         0    1.0000    1.0000   -0.0079    0.2443   -0.2443
         0         0    1.0000    1.0000    0.9921    0.8023   -0.8023
         0         0         0         0    1.0000    1.0000         0
>> [ntf,stf] = calculateTF(ABCD)
Zero/pole/gain:
       (z-1) (z^2 - 1.997z + 1) (z^2 - 1.992z + 1)
-----------------------------------------------------------
(z-0.7778) (z^2 - 1.613z + 0.6649) (z^2 - 1.796z + 0.8549)

Sampling time: 1

Zero/pole/gain:
1

Static gain.
```

realizeNTF_ct

Synopsis: `[ABCDc,tdac2] = realizeNTF_ct( ntf, form='FB', tdac, ordering=[1:n], bp=zeros(…), ABCDc)`

Realize a noise transfer function (NTF) with a continuous-time loop filter.

Arguments

ntf The modulator NTF, specified as an LTI object in zero-pole form.

form A string specifying the modulator topology.

- FB Feedback form.
- FF Feedforward form.

tdac The timing for the feedback DAC(s). If `tdac(1)`≥1, direct feedback terms are added to the quantizer. Multiple timings (one or more per integrator) for the FB topology can be specified by making tdac a cell array, e.g.
`tdac = {[1,2]; [1 2]; {[0.5 1],[1 1.5]}; []};`
In this example, the first two integrators have DACs with `[1,2]` timing, the third has a pair of dacs, one with `[0.5 1]` timing and the other with `[1 1.5]` timing, and there is no direct feedback DAC to the quantizer.

ordering A vector specifying which NTF zero-pair to use in each resonator. Default is for the zero-pairs to be used in the order specified in the NTF.

bp A vector specifying which resonator sections are bandpass. The default (zeros(...)) is for all sections to be lowpass.

ABCDc The loop filter structure, in state-space form. If this argument is omitted, ABCDc is constructed according to *form*.

Output

ABCDc A state-space description of the CT loop filter

tdac2 A matrix with the DAC timings, including ones that were automatically added.

Example

Realize the NTF $(1-z^{-1})^2$ with a CT system (cf. the example on page 15).

```
>> ntf = zpk([1 1],[0 0],1,1);
>> [ABCDc,tdac2] = realizeNTF_ct(ntf,'FB')
ABCDc =
         0         0    1.0000   -1.0000
    1.0000         0         0   -1.5000
         0    1.0000         0    0.0000
tdac2 =
    -1    -1
     0     1
```

mapCtoD

Synopsis: `[sys, Gp] = mapCtoD(sys_c,t=[0 1],f0=0)`
Map a MIMO continuous-time system to a SIMO discrete-time equivalent. The criterion for equivalence is that the sampled pulse response of the CT system must be identical to the impulse response of the DT system. I.e. if y_c is the output of the CT system with an input v_c taken from a set of DACs fed with a single DT input v, then y, the output of the equivalent DT system with input v satisfies $y(n) = y_c(n^-)$ for integer n. The DACs are characterized by rectangular impulse responses with edge times specified in the *t* matrix.

Arguments

sys_c	The LTI description of the CT system.
t	The edge times of the DAC pulse used to make CT waveforms from DT inputs. Each row corresponds to one of the system inputs; [-1 -1] denotes a CT input. The default is [0 1] for all inputs except the first, which is assumed to be a CT input.
f0	The (normalized) frequency at which the Gp filters' gains are to be set to unity. Default 0 (DC).

Output

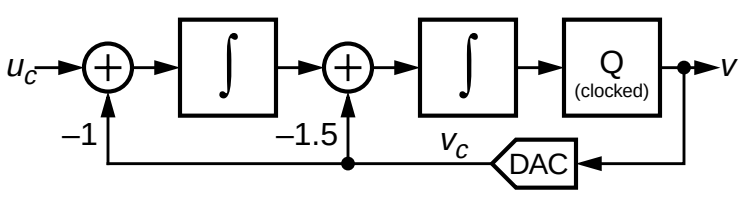
sys	The LTI description for the DT equivalent.
Gp	The mixed CT/DT prefilters which form the samples fed to each state for the CT inputs.

Reference

[1] R. Schreier and B. Zhang, "Delta-sigma modulators employing continuous-time circuitry," IEEE Transactions on Circuits and Systems I, vol. 43, no. 4, pp. 324-332, April 1996.

Example

Map the standard second-order CT modulator shown below to its DT equivalent and verify that the NTF. is $(1 - z^{-1})^2$



$$\begin{bmatrix} \dot{x}_{1c} \\ \dot{x}_{2c} \\ y_c \end{bmatrix} = \left[\begin{array}{cc|cc} 0 & 0 & 1 & -1 \\ 1 & 0 & 0 & -1.5 \\ \hline 0 & 1 & 0 & 0 \end{array}\right] \begin{bmatrix} \dot{x}_{1c} \\ \dot{x}_{2c} \\ u_c \\ v_c \end{bmatrix}$$

```
>> LFc = ss([0 0;1 0], [1 -1;0 -1.5], [0 1], [0 0]);
>> tdac = [0 1];
>> [LF,Gp] = mapCtoD(LFc,tdac);
>> ABCD = [LF.a LF.b; LF.c LF.d];
>> H = calculateTF(ABCD)

Zero/pole/gain:
(z-1)^2
-------
  z^2

Sampling time: 1
```

evalTFP

Synopsis: `H = evalTFP(Hs,Hz,f)`
Use this function to evaluate the signal transfer function of a continuous-time (CT) system. In this context `Hs` is the open-loop response of the loop filter from the *u* input and `Hz` is the closed-loop noise transfer function.

Arguments

Hs	A continuous-time transfer function in zpk form.
Hz	A discrete-time transfer function in zpk form.
f	A vector of frequencies.

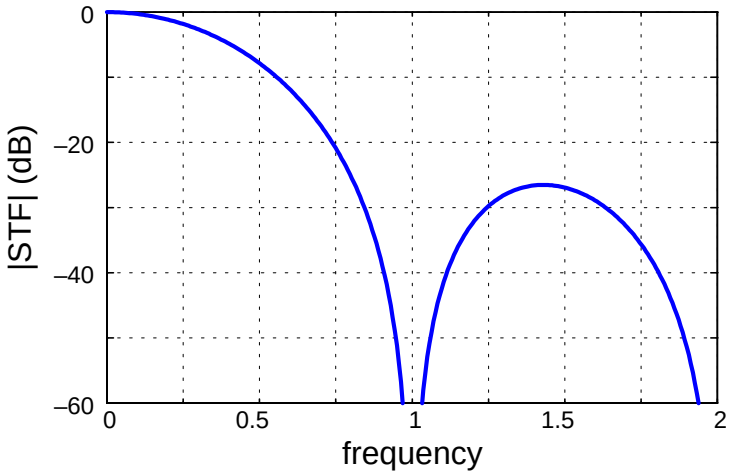
Output

H	The value of $H_s(j2\pi f)H_z(e^{j2\pi f})$. `evalTFP` attempts to cancel poles in H_s with zeros in H_z.

See Also
`evalMixedTF` is a more advanced version of this function which is used to evaluate the individual feed-in transfer functions of a CT modulator.

Example
Plot the STF of the 2nd-order CT system depicted on page 15.

```
Ac = [ 0 0; 1 0];       Bc = [1 -1; 0 -1.5];
Cc = [0 1];             Dc = [0 0];
LFc = ss(Ac, Bc, Cc, Dc);
L0c = zpk(ss(Ac,Bc(:,1),Cc,Dc(1)));
tdac = [0 1];
[LF,Gp] = mapCtoD(LFc,tdac);
ABCD = [LF.a LF.b; LF.c LF.d];
H = calculateTF(ABCD);
% Yields H=(1-z^-1)^2
f = linspace(0,2,300);
STF = evalTFP(L0c,H,f);
plot(f,dbv(STF));
```



simulateESL

Synopsis: `[sv,sx,sigma_se,max_sx,max_sy]`
`= simulateESL(v,mtf,M=16,dw=[1…], sx0=[0…])`

Simulate the element selection logic (ESL) of a multi-element DAC using a particular mismatch-shaping transfer function (`mtf`).

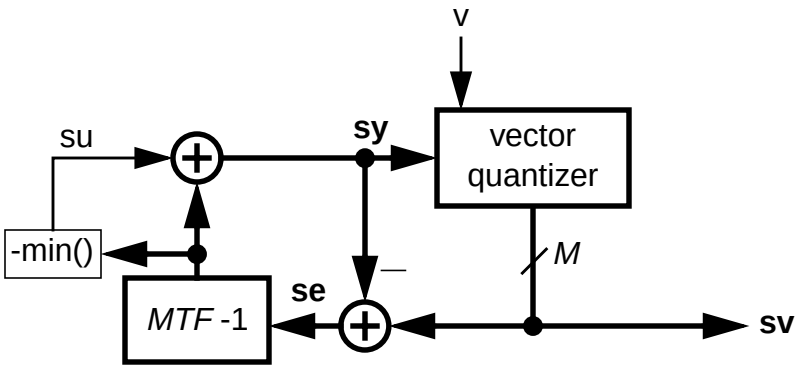
[1] R. Schreier and B. Zhang "Noise-shaped multibit D/A convertor employing unit elements," *Electronics Letters*, vol. 31, no. 20, pp. 1712-1713, Sept. 28 1995.

Arguments

v	A vector containing the number of elements to enable. Note that the output of `simulateDSM` must be offset and scaled in order to be used here as *v* must be in the range $[0, \sum_i^M dw(i)]$.
mtf	The mismatch-shaping transfer function, given in zero-pole form.
M	The number of elements.
dw	A vector containing the weight associated with each element.
sx0	An $n \times M$ matrix containing the initial state of the element selection logic.

Output

sv	The selection vector: a vector of zeros and ones indicating which elements to enable.
sx	An $n \times M$ matrix containing the final state of the element selection logic.
sigma_se	The rms value of the selection error, $\boldsymbol{se} = \boldsymbol{sv} - \boldsymbol{sy}$. *sigma_se* may be used to analytically estimate the power of in-band noise caused by element mismatch.
max_sx	The maximum value attained by any state in the ESL.
max_sy	The maximum value attained by any component of the (un-normalized) "desired usage" vector.

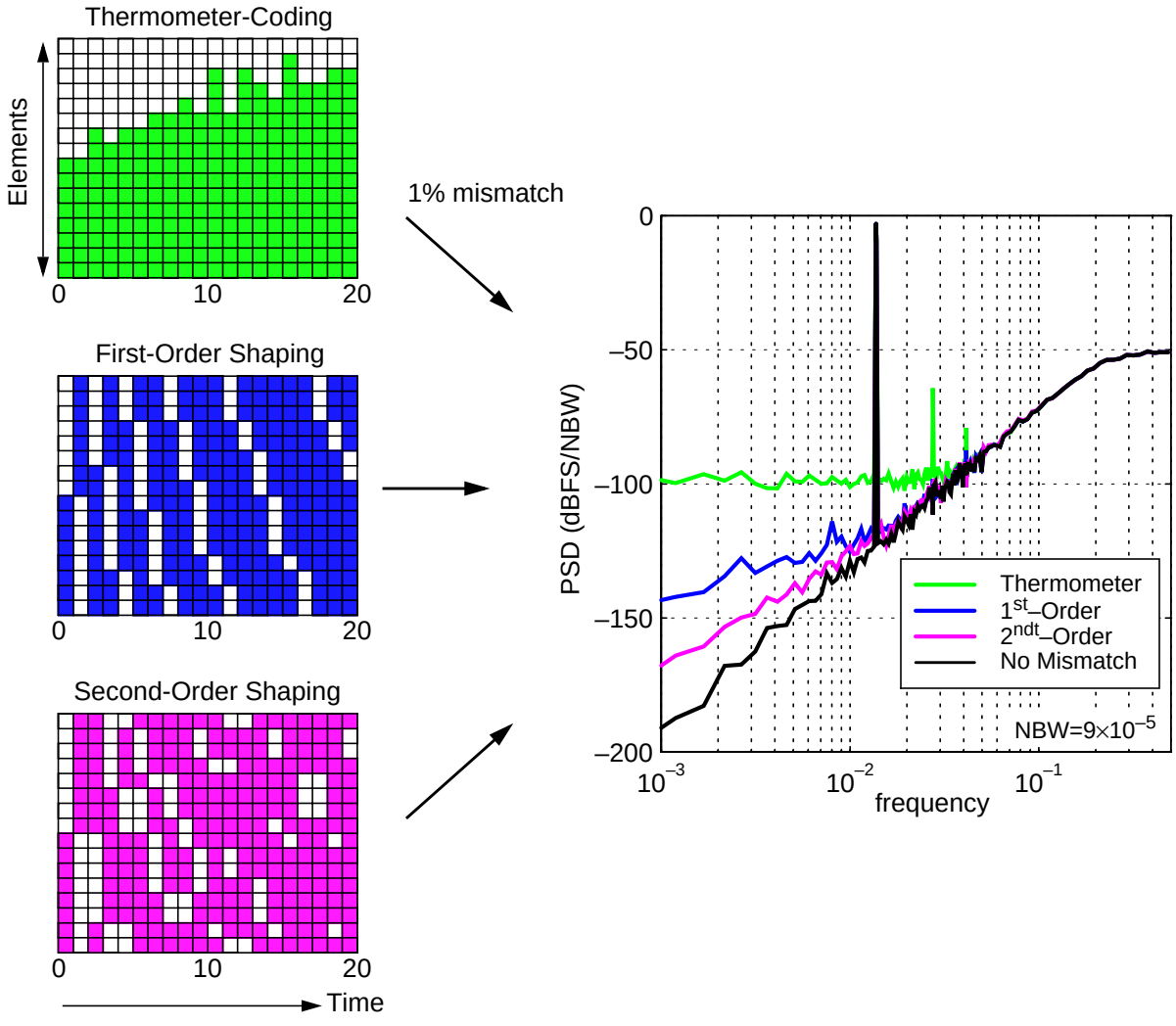


Block diagram of the Element Selection Logic

Example (From `dsdemo5`**)**

Compare the usage patterns and example spectra for a 16-element DAC driven with thermometer-coded, 1st-order mismatch-shaped and 2nd-order mismatch-shaped data generated by a 3rd-order multi-bit modulator.

```
>> ntf = synthesizeNTF(3,[],[],4);
>> M = 16;
>> N = 2^14;
>> fin = round(0.33*N/(2*12));
>> u = M/sqrt(2)*sin((2*pi/N)*fin*[0:N-1]);
>> v = simulateDSM(u,ntf,M+1);
>> v = (v+M)/2;                                    % scale v to [0,M]
>> sv0 = thermometer(v,M);
>> mtf1 = zpk(1,0,1,1);                           %First-order shaping
>> sv1 = simulateESL(v,mtf1,M);
>> mtf2 = zpk([ 1 1 ], [ 0 0 ], 1, 1);           %Second-order shaping
>> sv2 = simulateESL(v,mtf2,M);
>> ue = 1 + 0.01*randn(M,1);                             % 1% mismatch
>> dv0 = ue' * sv0;
>> spec0 = fft(dv0.*ds_hann(N))/(M*N/8);
>> plotSpectrum(spec0,fin,'g');
...
```



synthesizeQNTF

Synopsis: `ntf = synthesizeQNTF(order=3,OSR=64,f0=0,NG=-60,ING=-20,n_im=order/3)`
Synthesize a noise transfer function (NTF) for a quadrature delta-sigma modulator.

Arguments

order	The order of the NTF.
OSR	The oversampling ratio.
f0	The center frequency of the modulator. *f0*≠0 yields a bandpass modulator; `f0=0.25` puts the center frequency at $f_s/4$.
NG	The rms in-band noise gain (dB).
ING	The rms image-band noise gain (dB).
n_im	Number of image-band zeros.

Output

ntf	The modulator NTF, given as an LTI object in zero-pole form.

Bugs
ALPHA VERSION. This function uses an experimental ad hoc method that is neither optimal nor robust.

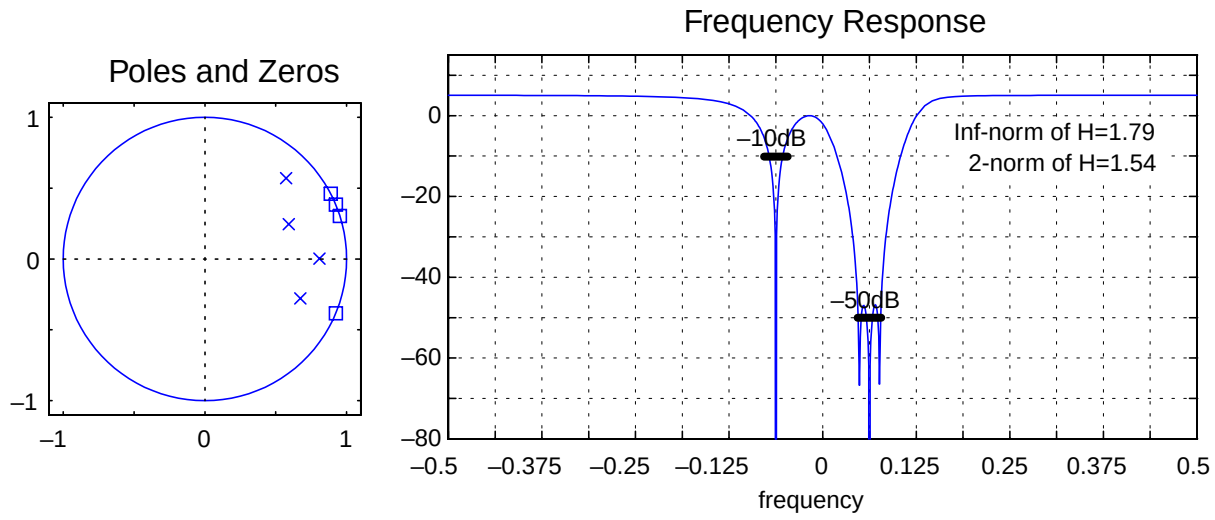
Example (From `dsexample4`**)**
Fourth-order, $OSR = 32$, $f_0 = 1/16$ bandpass NTF with an rms in-band noise gain of –50 dB and an image-band noise gain of –10 dB.

```
>> ntf = synthesizeQNTF(4,32,1/16,-50,-10);

Zero/pole/gain:

             (z-(0.953+0.3028i)) (z^2 - 1.848z + 1) (z-(0.888+0.4598i))
-------------------------------------------------------------------------------------
 (z-(0.8088+0.002797i)) (z-(0.5913+0.2449i)) (z-(0.6731-0.2788i)) (z-(0.5739+0.5699i))

Sampling time: 1
```



simulateQDSM

Synopsis: `[v,xn,xmax,y] = simulateQDSM(u,ABCD|ntf,nlev=2,x0=0)`
Simulate a quadrature delta-sigma modulator with a given input. For improved simulation speed, use `simulateDSM` with a 2-input/2-output ABCDr argument as indicated in the example below.

Arguments

u	The input sequence to the modulator, given as a $1 \times N$ row vector. Full-scale corresponds to an input of magnitude *nlev*–1.
ABCD	A state-space description of the modulator loop filter.
ntf	The modulator NTF, given in zero-pole form. The modulator STF is assumed to be unity.
nlev	The number of levels in the quantizer. Multiple quantizers are indicated by making *nlev* an array.
x0	The initial state of the modulator.

Output

v	The samples of the output of the modulator, one for each input sample.
xn	The internal states of the modulator, one for each input sample, given as an $n \times N$ matrix.
xmax	The maximum absolute values of each state variable.
y	The samples of the quantizer input, one per input sample.

Example

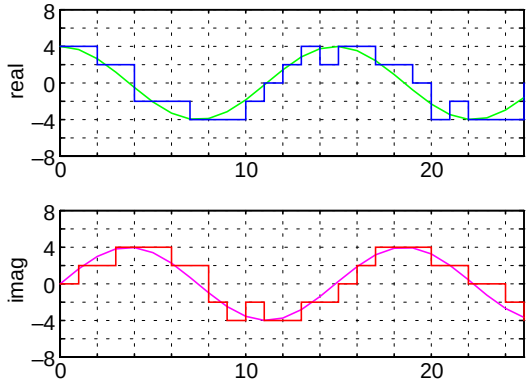
Simulate a 4th-order 9-level $f_0 = 1/16$ quadrature modulator with a half-scale sine-wave input and plot its output in the time and frequency domains.

```
nlev = 9; f0 = 1/16; osr = 32; M = nlev-1;
ntf = synthesizeQNTF(4,osr,f0,-50,-10);
N = 64*osr; f = round((f0+0.3*0.5/osr)*N)/N;
u = 0.5*M*exp(2i*pi*f*[0:N-1]);
v = simulateQDSM(u,ntf,nlev);
```

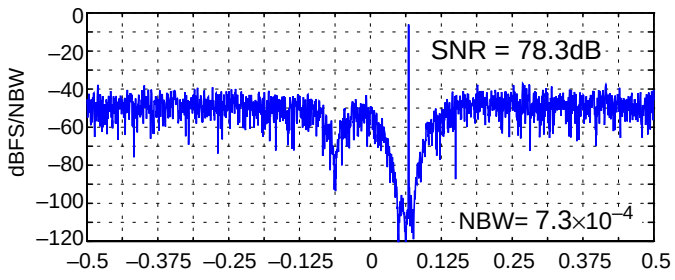
faster code

```
ABCD = realizeQNTF(ntf,'FF');
ABCDr = mapQtoR(ABCD);
ur = [real(u); imag(u)];
vr=simulateDSM(ur,ABCDr,nlev*[1;1]);
v = vr(1,:) + 1i*vr(2,:);
```

```
t = 0:25;
subplot(211)
plot(t, real(u(t+1)),'g');
hold on;
stairs(t,real(v(t+1)),'b');
figureMagic(…)
ylabel('real');
```




```
spec = fft(v.*ds_hann(N))/(M*N/2);
spec = [fftshift(spec) spec(N/2+1)];
plot(linspace(-0.5,0.5,N+1), dbv(spec))
figureMagic([-0.5 0.5],1/16,2, [-120 0],10,2)
ylabel('dBFS/NBW')
[f1 f2] = ds_f1f2(osr,f0,1);
fb1 = round(f1*N); fb2 = round(f2*N);
fb = round(f*N)-fb1;
snr = calculateSNR(spec(N/2+1+[fb1:fb2]),fb);
text(f,-10,sprintf(' SNR = %4.1fdB\n',snr));
text(0.25, -105, sprintf('NBW=%0.1e',1.5/N));
```



realizeQNTF

Synopsis: `ABCD = realizeQNTF(ntf,form='FB',rot=0,bn)`
Convert a quadrature noise transfer function (NTF) into an ABCD matrix for the desired structure.

Arguments

ntf	The modulator NTF, given in zero-pole form (i.e. a zpk object).
form	A string specifying the modulator topology.
	FB Feedback
	PFB Parallel feedback
	FF Feedforward
	PFF Parallel feedforward
	FBD FB with delaying quantizer. NOT SUPPORTED YET
	FFD FF with delaying quantizer. NOT SUPPORTED YET
rot	*rot* =1 means rotate states to make as many coefficients as possible real
bn	The coefficient of the auxiliary DAC for *form* = 'FF'

Output

ABCD	State-space description of the loop filter.

Example (From `dsexample4`**)**
Determine coefficients for the parallel feedback (PFB) structure,.

```
>> ntf = synthesizeQNTF(5,32,1/16,-50,-10);
>> ABCD = realizeQNTF(ntf,'PFB',1)
ABCD =
  Columns 1 through 4
   0.8854 + 0.4648i        0                  0                  0
   0.0065 + 1.0000i   0.9547 + 0.2974i        0                  0
        0             0.9715 + 0.2370i   0.9088 + 0.4171i        0
        0                  0             0.8797 + 0.4755i   0.9376 + 0.3477i
        0                  0                  0                  0
        0                  0                  0            -0.9916 - 0.1294i
  Columns 5 through 7
        0             0.0025             0.0025 + 0.0000i
        0                  0             0.0262 + 0.0000i
        0                  0             0.1791 + 0.0000i
        0                  0             0.6341 + 0.0000i
   0.9239 - 0.3827i        0             0.1743 + 0.0000i
  -0.9312 - 0.3645i        0                  0
```

mapQtoR

Synopsis: `ABCDr = mapQtoR(ABCD)`
Convert a quadrature matrix into its real (IQ) equivalent.

Arguments

ABCD A complex matrix describing a quadrature system.

Output

ABCDr A real matrix corresponding to ABCD. Each element *z* in ABCD is replaced by a 2×2 matrix in ABCDr. Specifically

$$z \rightarrow \begin{bmatrix} x & -y \\ y & x \end{bmatrix} \text{ where } x = Re(z) \text{ and } y = Im(z).$$

mapRtoQ

Synopsis: `[ABCDq ABCDp] = mapR2Q( ABCDr )`
Map a real `ABCD` to a quadrature `ABCD`. `ABCDr` has its states paired (real, imaginary).

Arguments

ABCDr A real matrix describing a quadrature system.

Output

ABCDq The quadrature (complex) version of `ABCDr`.

ABCDp The mirror-image system matrix. `ABCDp` is zero if `ABCDr` has no quadrature errors.

calculateQTF

Synopsis: `[ntf stf intf istf] = calculateQTF(ABCDr)`
Calculate the noise and signal transfer functions for a quadrature modulator.

Arguments

ABCDr A real state-space description of the modulator loop filter. I/Q asymmetries may be included in the description. These asymmetries result in non-zero image transfer functions.

Output

ntf, stf The noise and signal transfer functions.

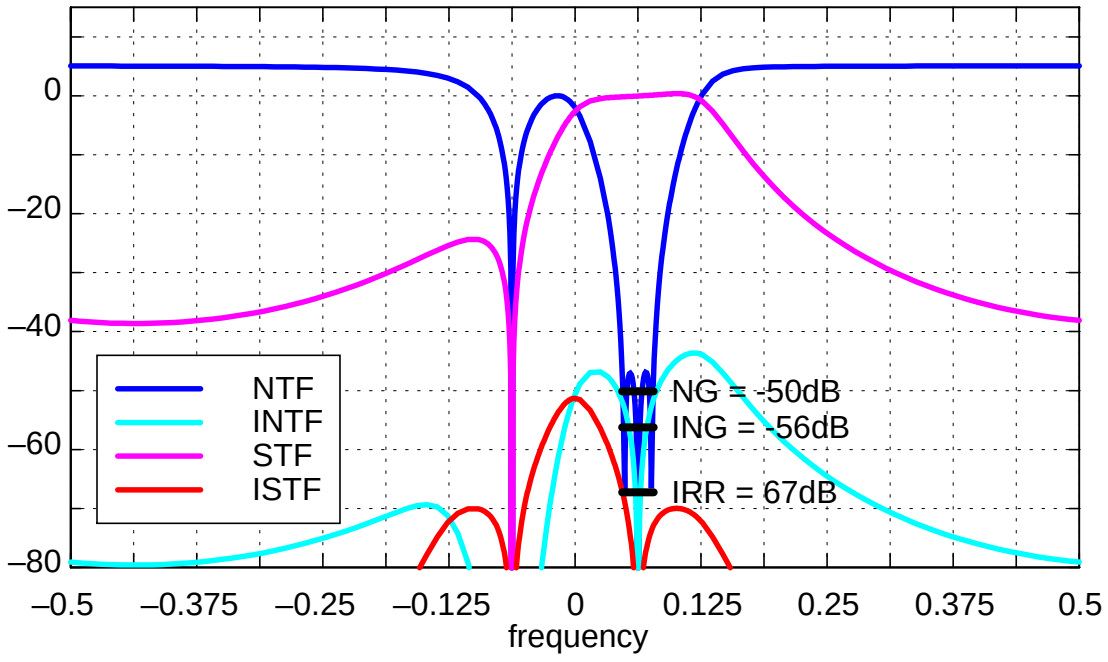
intf, istf The image noise and image signal transfer functions.

All transfer functions are returned as LTI systems in zero-pole form.

Example (Continuing from `dsexample4`)

Examine the effect of mismatch in the first feedback.

```
>> ABCDr = mapQtoR(ABCD);
>> ABCDr(2,end) = 1.01*ABCDr(2,end);   % 0.1% mismatch in first feedback
>> [H G HI GI] = calculateQTF(ABCDr);
```



simulateQESL

Synopsis: `[sv,sx,sigma_se,max_sx,max_sy] = simulateQESL(v,mtf,M=16,sx0=[0…])`
Simulate the element selection logic (ESL) for a quadrature differential DAC.

Arguments

v	A vector the digital input values.
mtf	The mismatch-shaping transfer function, given in zero-pole form.
M	The number of elements. There is a total $2M$ elements.
sx0	An $n \times M$ matrix whose columns are the initial state of the ESL.

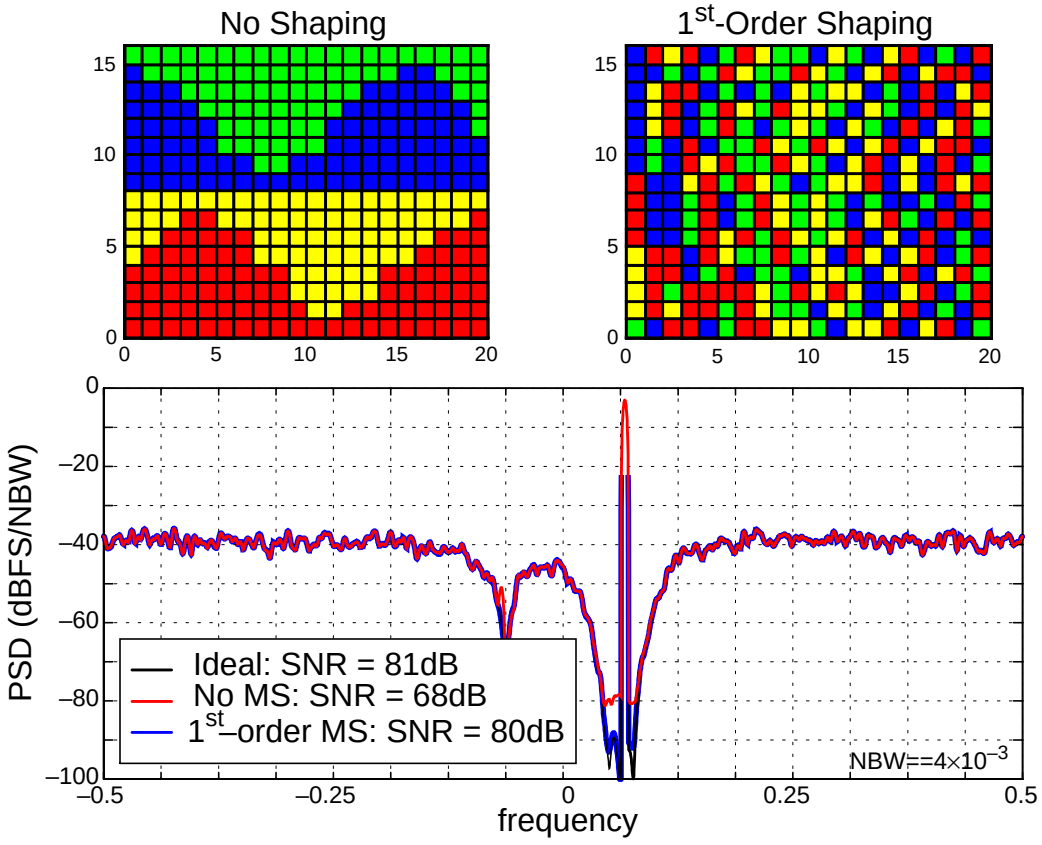
Output

sv	The selection vector: a vector of zeros and ones indicating which elements to enable.
sx	An $n \times M$ matrix containing the final state of the ESL.
sigma_se	The rms value of the selection error, $\boldsymbol{se} = \boldsymbol{sv} - \boldsymbol{sy}$. *sigma_se* may be used to analytically estimate the power of in-band noise caused by element mismatch.
max_sx	The maximum absolute value attained by any state in the ESL.
max_sy	The maximum absolute value attained by any component of the input to the VQ.

Example

Apply mismatch-shaping to the modulator from `dsexample4`**.**

```
>> mtf1 = zpk(exp(2i*pi*f0),0,1,1);          %First-order complex shaping
>> sv1 = simulateQESL(v,mtf1,M);
```



designHBF

Synopsis: `[f1,f2,info]=designHBF(fp=0.2,delta=1e-5,debug=0)`
Design a hardware-efficient linear-phase half-band filter for use in the decimation or interpolation filter associated with a delta-sigma modulator. This function is based on the procedure described by Saramäki [1]. Note that since the algorithm uses a non-deterministic search procedure, successive calls may yield different designs.

[1] T. Saramäki, "Design of FIR filters as a tapped cascaded interconnection of identical subfilters," *IEEE Transactions on Circuits and Systems*, vol. 34, pp. 1011-1029, 1987.

Arguments

fp	Normalized passband cutoff frequency.
delta	Passband and stopband ripple in absolute value.

Output

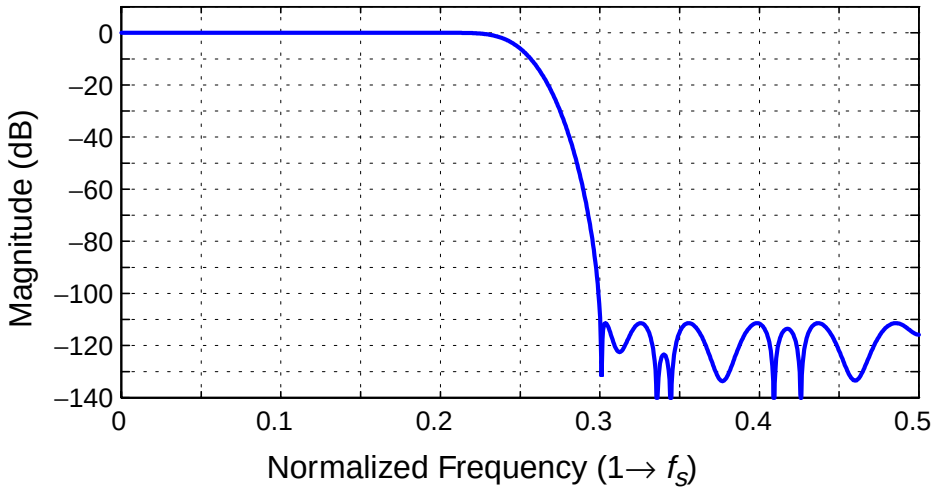
f1,f2	Prototype filter and subfilter coefficients and their canonical-signed digit (csd) representation.
info	A vector containing the following information data (only set when `debug=1`):

complexity	The number of additions per output sample.
n1,n2	The length of the f1 and f2 vectors.
sbr	The achieved stop-band attenuation (dB).
phi	The scaling factor for the F2 filter.

Example
Design of a lowpass half-band filter with a cut-off frequency of $0.2f_s$, a passband ripple of less than 10^{-5} and a stopband rejection of at least 10^{-5} (-100 dB).

```
>> [f1,f2] = designHBF(0.2,1e-5);
>> f = linspace(0,0.5,1024);
>> plot(f, dbv(frespHBF(f,f1,f2)))
```

A plot of the filter response is shown below. The filter achieves 109 dB of attenuation in the stopband and uses only 124 additions (no true multiplications) to produce each output sample.



The structure of this filter as a decimation or interpolation filter is shown below. The coefficients and their signed-digit decompositions are

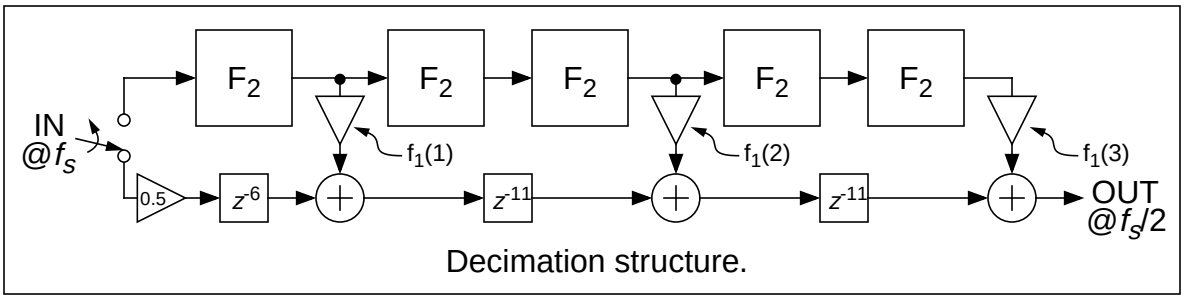
```
[f1.val]' =
    0.9453
   -0.6406
    0.1953
```

```
[f2.val]' =
    0.6211
   -0.1895
    0.0957
   -0.0508
    0.0269
   -0.0142
```

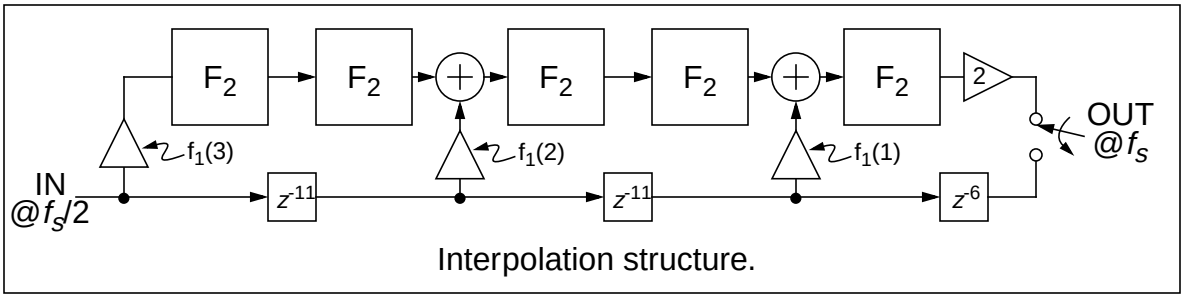
```
>> f1.csd
ans =
     0    -4    -7
     1    -1     1
ans =
    -1    -3    -6
    -1    -1    -1
ans =
    -2    -4    -7
     1    -1     1
```

```
>> f2.csd
ans =
    -1    -3    -8
     1     1    -1
ans =
    -2    -4    -9
    -1     1    -1
ans =
    -3    -5    -9
     1    -1     1
ans =
    -4    -7    -8
    -1     1     1
ans =
    -5    -8   -11
     1    -1    -1
ans =
    -6    -9   -11
    -1     1    -1
```

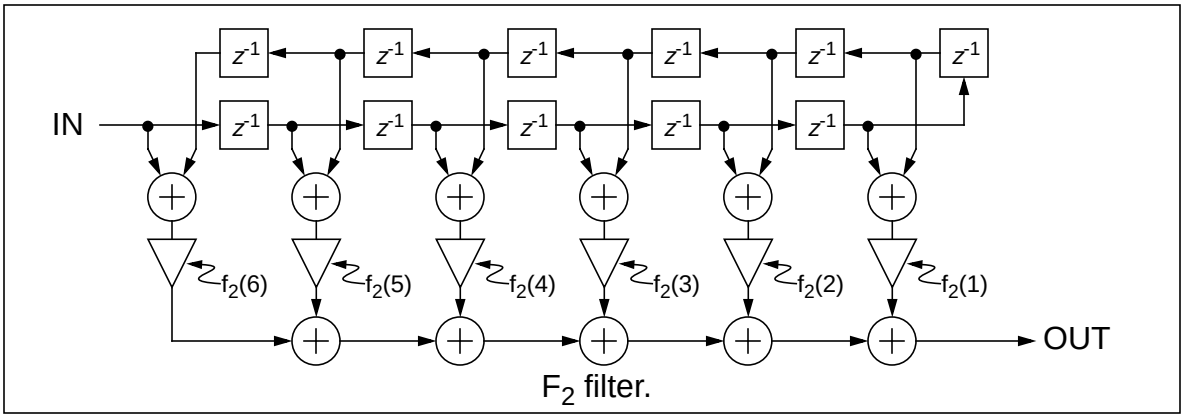
In the signed-digit expansions, the first row contain the powers of two while the second row gives their signs. For example, $f_1(1) = 0.9453 = 2^0-2^{-4}+2^{-7}$ and $f_2(1) = 0.6211 = 2^{-1}+2^{-3}-2^{-8}$. Since the filter coefficients for this example use only 3 signed digits, each multiply-accumulate operation shown in the diagram below needs only 3 binary additions. Thus, an implementation of this 110th-order FIR filter needs to perform only $3\times3 + 5\times(3\times6 + 6 -1) = 124$ additions at the low ($f_s/2$) rate.



Decimation structure.



Interpolation structure.



F2 filter.

simulateHBF

Synopsis: `y = simulateHBF(x,f1,f2,mode=0)`
Simulate a Saramaki half-band filter (see `designHBF` on page 25) in the time domain.

Arguments

x	The input data.
f1,f2	Filter coefficients. f1 and f2 can be vectors of values or struct arrays like those returned from `designHBF`.
mode	The mode flag determines whether the input is filtered, interpolated, or decimated according to the following: 0 Plain filtering, no interpolation or decimation. 1 The input is interpolated 2 The output is decimated, even samples are taken. 3 The output is decimated, odd samples are taken.

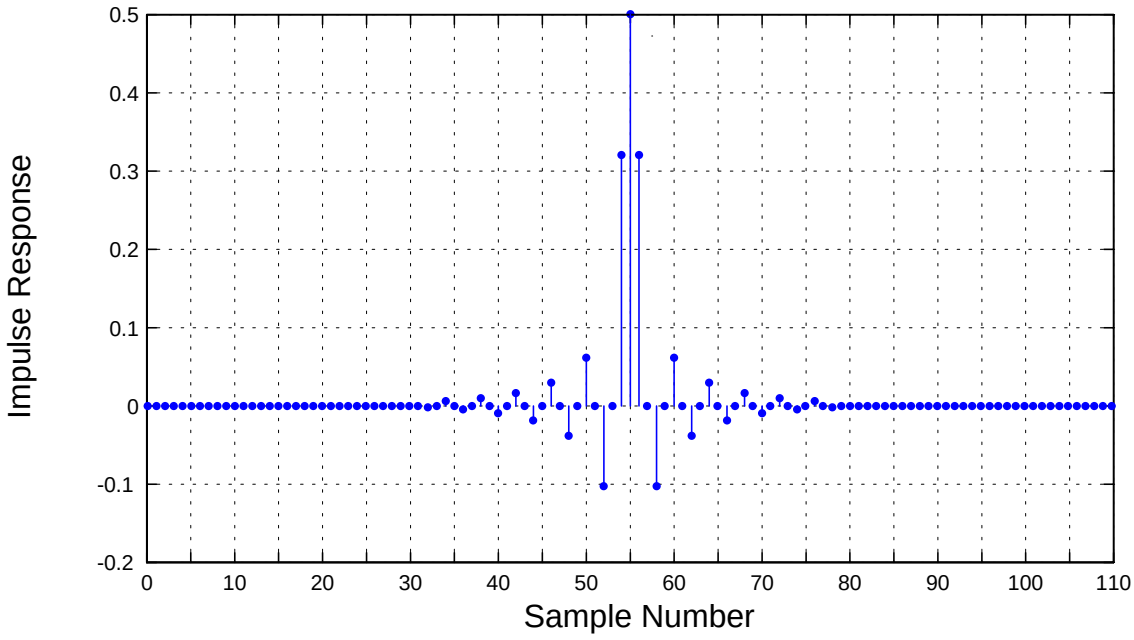
Output

y	The output data.

Example

Plot the impulse response of the HBF designed on the previous page.

```
>> N = (2*length(f1)-1)*2*(2*length(f2)-1)+1;
>> y = simulateHBF([1 zeros(1,N-1)],f1,f2);
>> stem([0:N-1],y);
>> figureMagic([0 N-1],5,2, [-0.2 0.5],0.1,1)
>> printmif('HBFimp', [6 3], 'Helvetica8')
```



predictSNR

Synopsis: `[snr,amp,k0,k1,sigma_e2] = predictSNR(ntf,OSR=64,amp=...,f0=0)`
Use the describing function method of Ardalan and Paulos [1] to predict the signal-to-noise ratio (SNR) in dB for various input amplitudes. This method is only applicable to binary modulators.

[1] S. H. Ardalan and J. J. Paulos, "Analysis of nonlinear behavior in delta-sigma modulators," *IEEE Transactions on Circuits and Systems*, vol. 34, pp. 593-603, June 1987.

Arguments

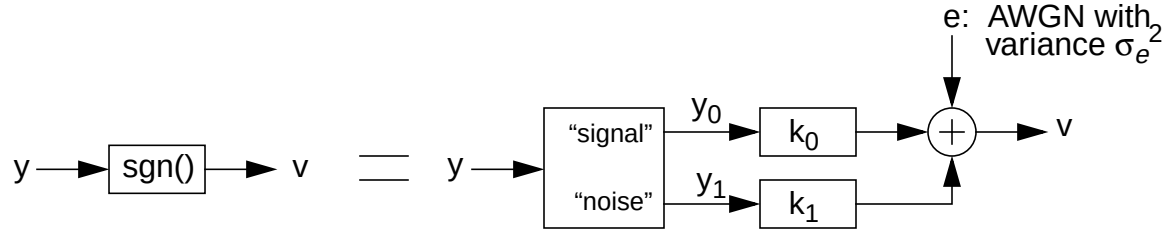
ntf	The modulator NTF, given in zero-pole form.
OSR	The oversampling ratio. *OSR* is used to define the "band of interest."
amp	A row vector listing the amplitudes to use. Defaults to [-120 -110...-20 -15 -10 -9 -8 … 0] dB, where 0 dB means a full-scale (peak value = 1) sine wave.
f0	The center frequency of the modulator. *f0*≠0 corresponds to a bandpass modulator.

Output

snr	A row vector containing the predicted SNRs.
amp	A row vector listing the amplitudes used.
k0	The signal gain of the quantizer model; one value per input level.
k1	The noise gain of the quantizer model; one value per input level.
sigma_e2	The mean square value of the noise in the model of the quantizer.

Example
See the example on page 9.

The Quantizer Model:
The binary quantizer is modeled as a pair of linear gains and a noise source, as shown in the figure below. The input to the quantizer is divided into signal and noise components which are processed by signal-dependent gains k_0 and k_1. These signals are added to a noise source, which is assumed to be white and to have a Gaussian distribution (the variance σ_e^2 is also signal-dependent), to produce the quantizer output.



findPIS, find2dPIS (in the `PosInvSet` subdirectory)

Synopsis:
```
[s,e,n,o,Sc] = findPIS(u,ABCD,nlev=2,options)
options = [dbg=0 itnLimit=2000 expFactor=0.005 N=1000 skip=100
           qhullArgA=0.999 qhullArgC=.001]
s = find2dPIS(u,ABCD,options)
options = [dbg=0 itnLimit=100 expFactor=0.01 N=1000 skip=100]
```
Find a convex positively-invariant set for a delta-sigma modulator. `findPIS` requires compilation of the `qhull mex` file; `find2dPIS` does not, but is limited to second-order systems.

Arguments

u	The input to the modulator. If *u* is a scalar, the input to the modulator is constant. If *u* is a 2×1 vector, the input to the modulator may be any sequence whose samples lie in the range $[u(1), u(2)]$.
ABCD	A state-space description of the modulator loop filter.
nlev	The number of quantizer levels.
dbg	Set dbg=1 to get a graphical display of the iterations.
itnLimit	The maximum number of iterations.
expFactor	The expansion factor applied to the hull before every mapping operation. Increasing expFactor decreases the number of iterations but results in sets which are larger than they need to be.
N	The number of points to use when constructing the initial guess.
skip	The number of time steps to run the modulator before observing the state. This handles the possibility of “transients” in the modulator.
qhullArgA	The ‘A’ argument to the qhull program. Adjacent facets are merged if the cosine of the angle between their normals is greater than the absolute value of this parameter. Negative values imply that the merge operation is performed during hull construction, rather than as a post-processing step.
qhullArgC	The ‘C’ argument to the qhull program. A facet is merged into its neighbor if the distance between the facet’s centrum (the average of the facet’s vertices) and the neighboring hyperplane is less than the absolute value of this parameter. As with the above argument, negative values imply pre-merging while positive values imply post-merging.

Output

s	The vertices of the set ($dim \times n_v$)).
e	The edges of the set, listed as pairs of vertex indices ($2 \times n_e$).
n	The normals for the facets of the set ($dim \times n_f$).
o	The offsets for the facets of the set ($1 \times n_f$).
Sc	The scaling matrix which was used internally to “round out” the set.

Background

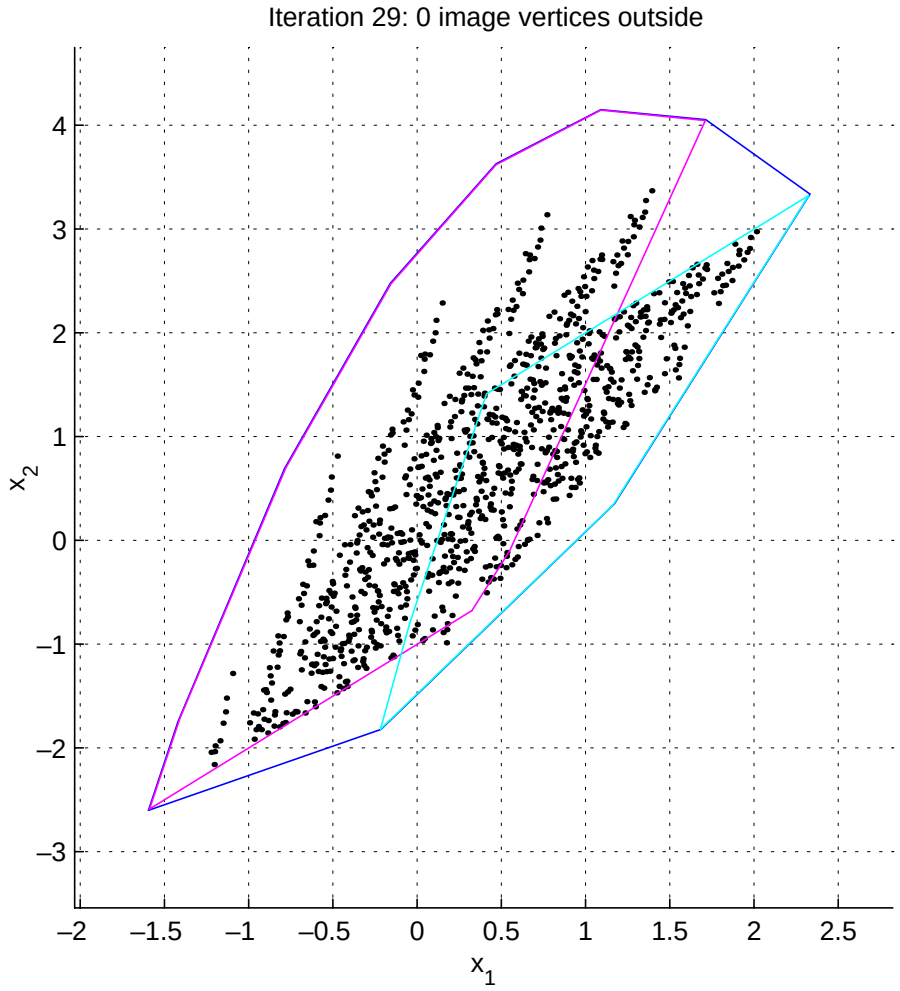
This is an implementation of the method described in [1].

[1] R. Schreier, M. Goodson and B. Zhang “An algorithm for computing convex positively invariant sets for delta-sigma modulators,” *IEEE Transactions on Circuits and Systems I*, vol. 44, no. 1, pp. 38-44, January 1997.

Example

Find a positively-invariant set for the second-order modulator with an input of $1/\sqrt{7}$.

```
>> ABCD = [
     1     0     1    -1
     1     1     1    -2
     0     1     0     0];
>> s = find2dPIS(sqrt(1/7),ABCD,1)
s =
  Columns 1 through 7
   -1.5954   -0.2150    1.1700    2.3324    1.7129    1.0904    0.4672
   -2.6019   -1.8209    0.3498    3.3359    4.0550    4.1511    3.6277
  Columns 8 through 11
   -0.1582   -0.7865   -1.4205   -1.5954
    2.4785    0.6954   -1.7462   -2.6019
```



designLCBP

Synopsis: `[param,H,L0,ABCD,x]=`
`designLCBP(n=3,OSR=64,opt=2,Hinf=1.6,f0=1/4,t=[0 1],form='FB',x0,dbg)`

Design a continuous-time LC-bandpass modulator consisting of resistors and LC tanks driven by transconductors and current-source DACs. Dynamic range and impedance scaling are not applied.

Arguments

n	Number of LC tanks in the loop filter.
OSR	Oversampling ratio, $OSR = f_s/(2f_b)$
opt	A flag indicating whether optimized NTF zeros should be used or not.
H_inf	The maximum out-of-band gain of the NTF. See `synthesizeNTF` on page 5.
f0	Modulator center frequency, default = $f_s/4$.
t	Start and end times of the DAC feedback pulse. Note that $t_1 = 0$ implies the use of a comparator with zero delay, which is usually an impractical situation.
form	The specific form of the modulator. See the table and diagram below.
dbg	Set this to a non-zero value to observe the optimization process in action.

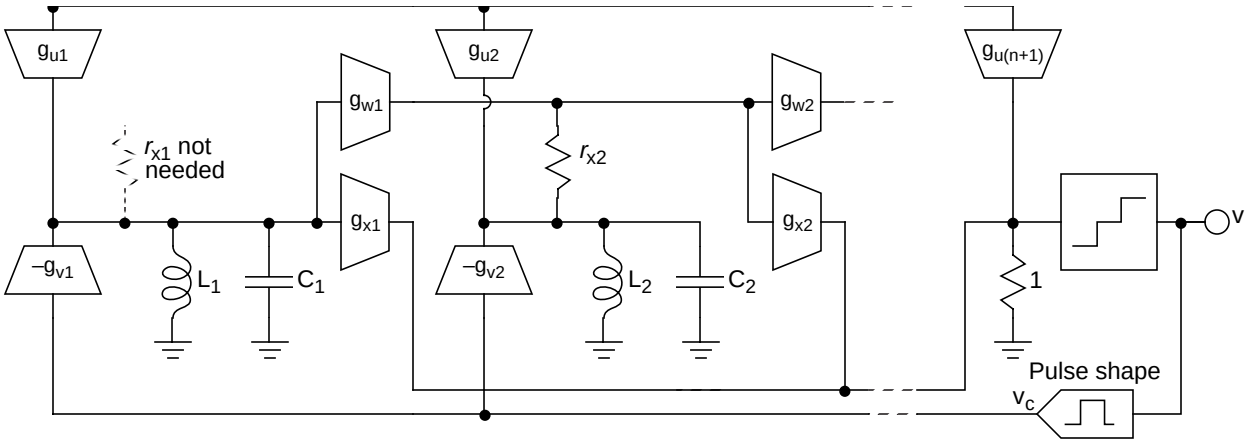
Output

param	A struct containing the (`n`, `OSR`, `Hinf`, `f0`, `t` and `form`) arguments plus the fields given in the table below.
H	The NTF of the equivalent discrete-time modulator.
L0	The continuous-time loop filter; an LTI object in zero-pole form.

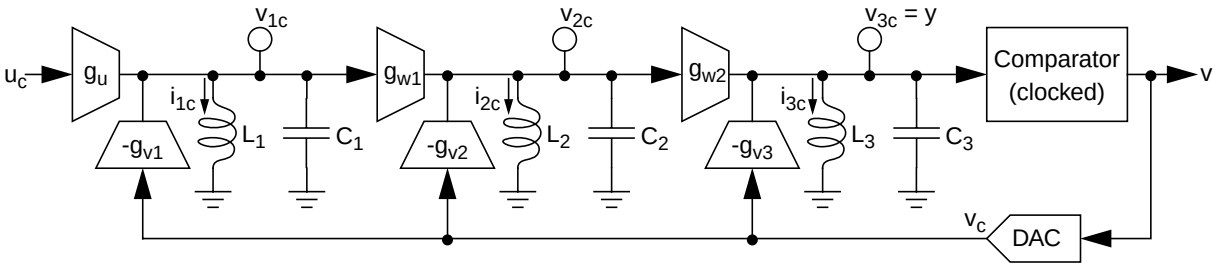
General LC topology and the coefficients subject to optimization for each of the supported forms:

Form	L	C	g_u: $1\times(n+1)$	g_v: $1\times n$	g_w: $1\times(n-1)$	g_x: $1\times n$	r_x: $1\times n$
FB	$\frac{1}{\sqrt{2\pi f_0}}$		[1 0 …]*	[x_1 x_2… x_n]	[1…]	[0 0… 1]	[0 …]
FF			[1 0 … 1]*	[1 0 …]	[1…]	[x_1 x_2… x_n]	[0 …]
R			[1 0 … 1]*	[x_1 0 …]	[1…]	[0 0… 1]	[0 x_2…x_n]

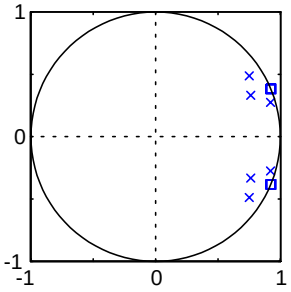
* scaled for unity STF gain at f_0.



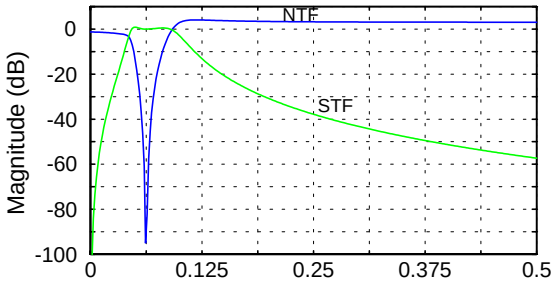
Example three-tank LC bandpass modulator (FB structure)



NTF Pole-Zero Plot



NTF and STF Frequency Response



Example

```
>> n = 3; OSR = 64; opt = 0; Hinf = 1.6; f0 = 1/16; t = [0.5 1]; form = 'FB';
>> [param,H,L0,ABCD,x] = designLCBP(n,OSR,opt,Hinf,f0,t,form);
>> plotPZ(H);
>> f = linspace(0,0.5,300); z = exp(2*pi*j*f);
>> ntf = dbv(evalTF(H,z)); stf = dbv(evalTFP(L0,H,f));
>> plot( f, ntf,'b', f, stf,'g');
>> figureMagic( [0,0.5],1/16,2, [-100 10],10,2);
gu = 1     0     0     0
gv = 0.653 2.703 3.708
gw = 1     1
gx = 0     0     1
rx = 0     0     0
```

See Also

`[H,L0,ABCD,k]=LCparam2tf(param,k=1)`

This function computes the transfer functions, quantizer gain and ABCD representation of an LC system. Inductor series resistance (`rl`) and capacitor shunt conductance (`gc`) can be incorporated into the calculations. Finite input and output conductance of the transconductors can be lumped with `gc`.

Bugs

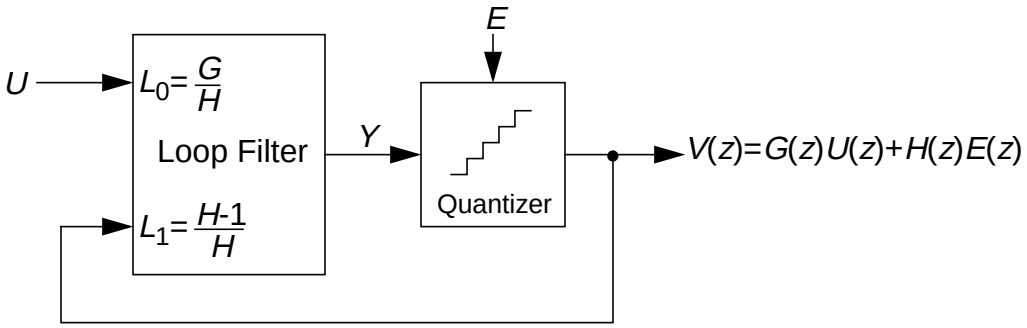
The use of the `constr/fmincon` function (from the optimization toolbox, versions 5 & 6) makes convergence of `designLCBP` erratic and unreliable. In some cases, editing the `LCObj*` functions helps. A more robust optimizer/objective function, perhaps one which supports a step-size restriction, is needed.

`designLCBP` is outdated, now that LC modulators which use more versatile stages in the back-end have been developed. See [1] for an example design.

[1] R. Schreier, J. Lloyd, L. Singer, D. Paterson, M. Timko, M. Hensley, G. Patterson, K. Behel, J. Zhou and W. J, Martin, "A 10-300 MHz IF-digitizing IC with 90-105 dB dynamic range and 15-333 kHz bandwidth," *IEEE Journal of Solid-State Circuits*, vol. SC-37, no. 12, pp. 1636-1644, Dec. 2002.

MODULATOR MODEL DETAILS

A delta-sigma modulator with a single quantizer is assumed to consist of quantizer connected to a loop filter as shown in the diagram below.



The Loop Filter

The loop filter is described by an ABCD matrix. For single-quantizer systems, the loop filter is a two-input, one-output linear system and ABCD is an $(n+1)\times(n+2)$ matrix, partitioned into A $(n\times n)$, B $(n\times 2)$, C $(1\times n)$ and D (1×2) sub-matrices as shown below:

$$ABCD = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right]. \tag{1}$$

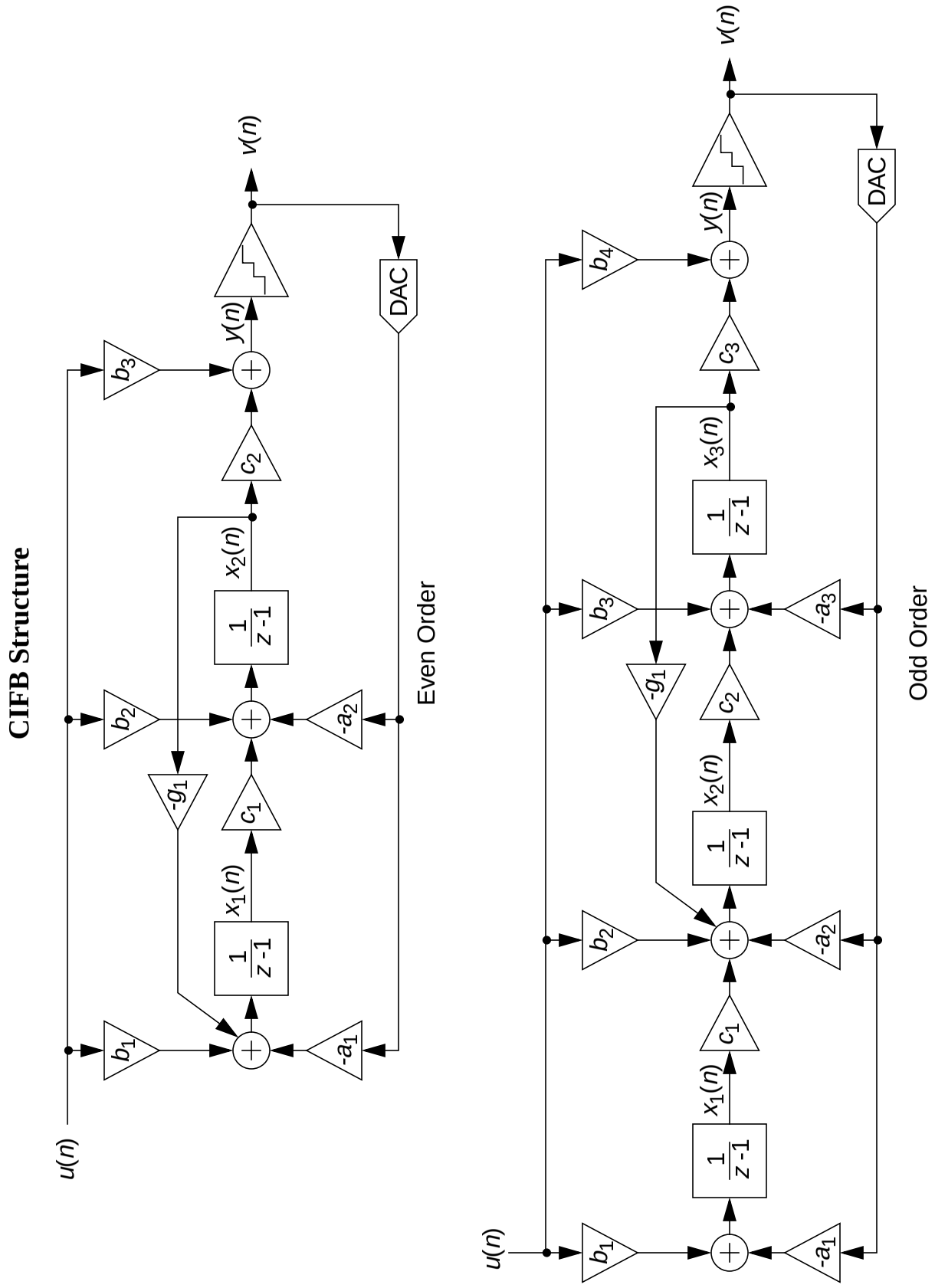
The equations for updating the state and computing the output of the loop filter are

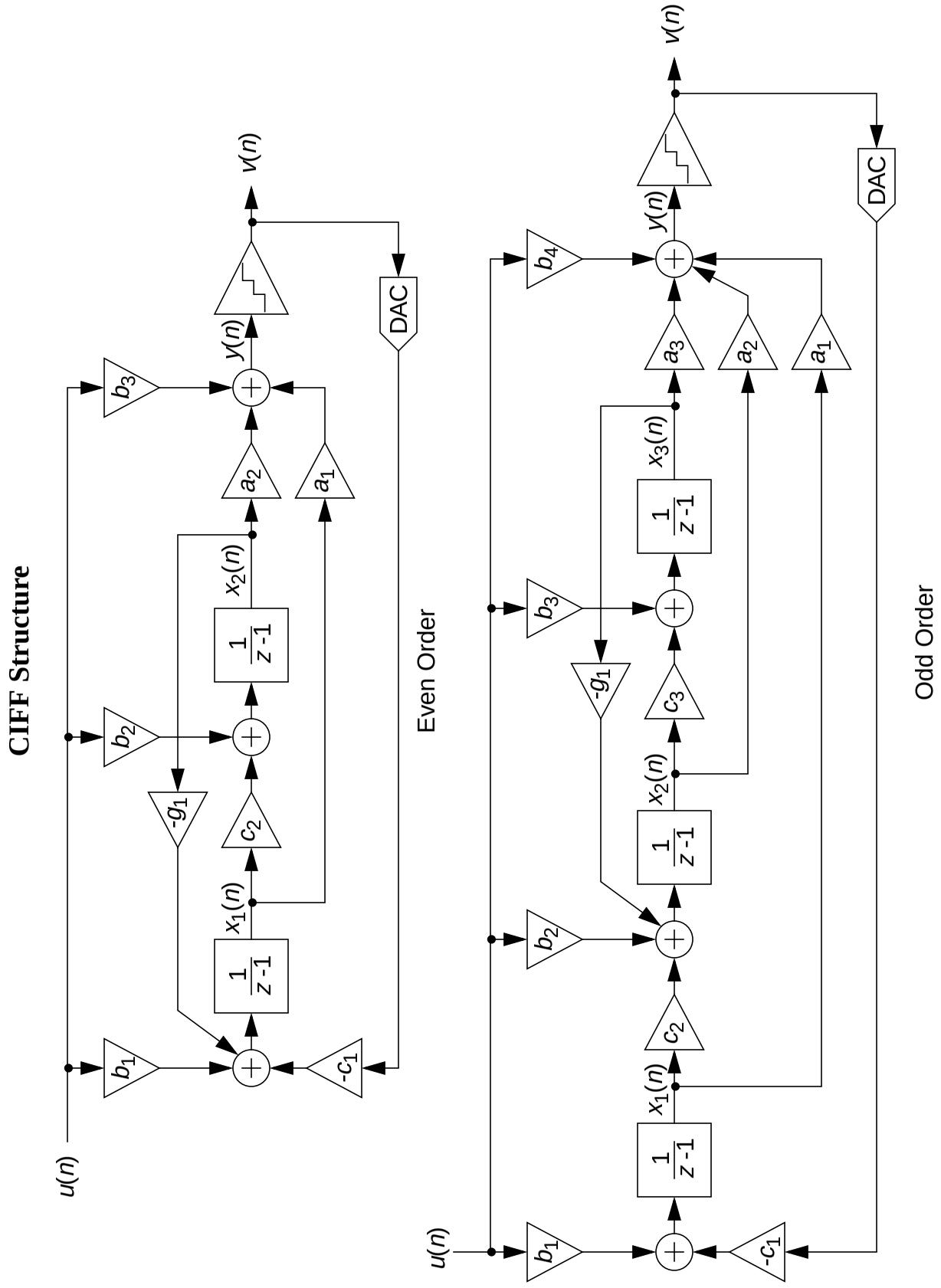
$$\begin{aligned} x(n+1) &= Ax(n) + B\begin{bmatrix} u(n) \\ v(n) \end{bmatrix} \\ y(n) &= Cx(n) + D\begin{bmatrix} u(n) \\ v(n) \end{bmatrix}. \end{aligned} \tag{2}$$

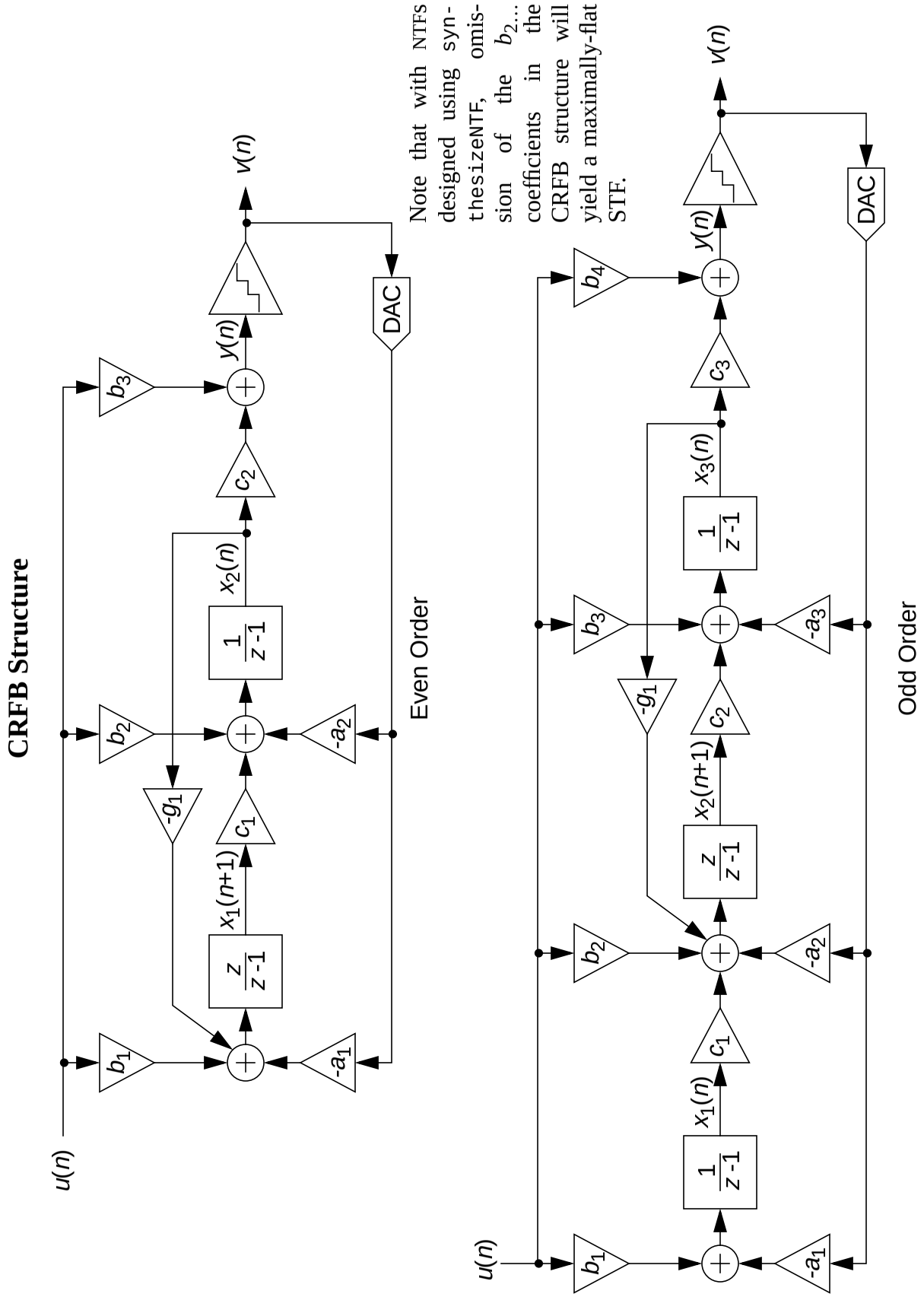
This formulation is sufficiently general to encompass all single-quantizer modulators which employ linear loop filters. The toolbox currently supports translation to/from an ABCD description and coefficients for the following topologies:

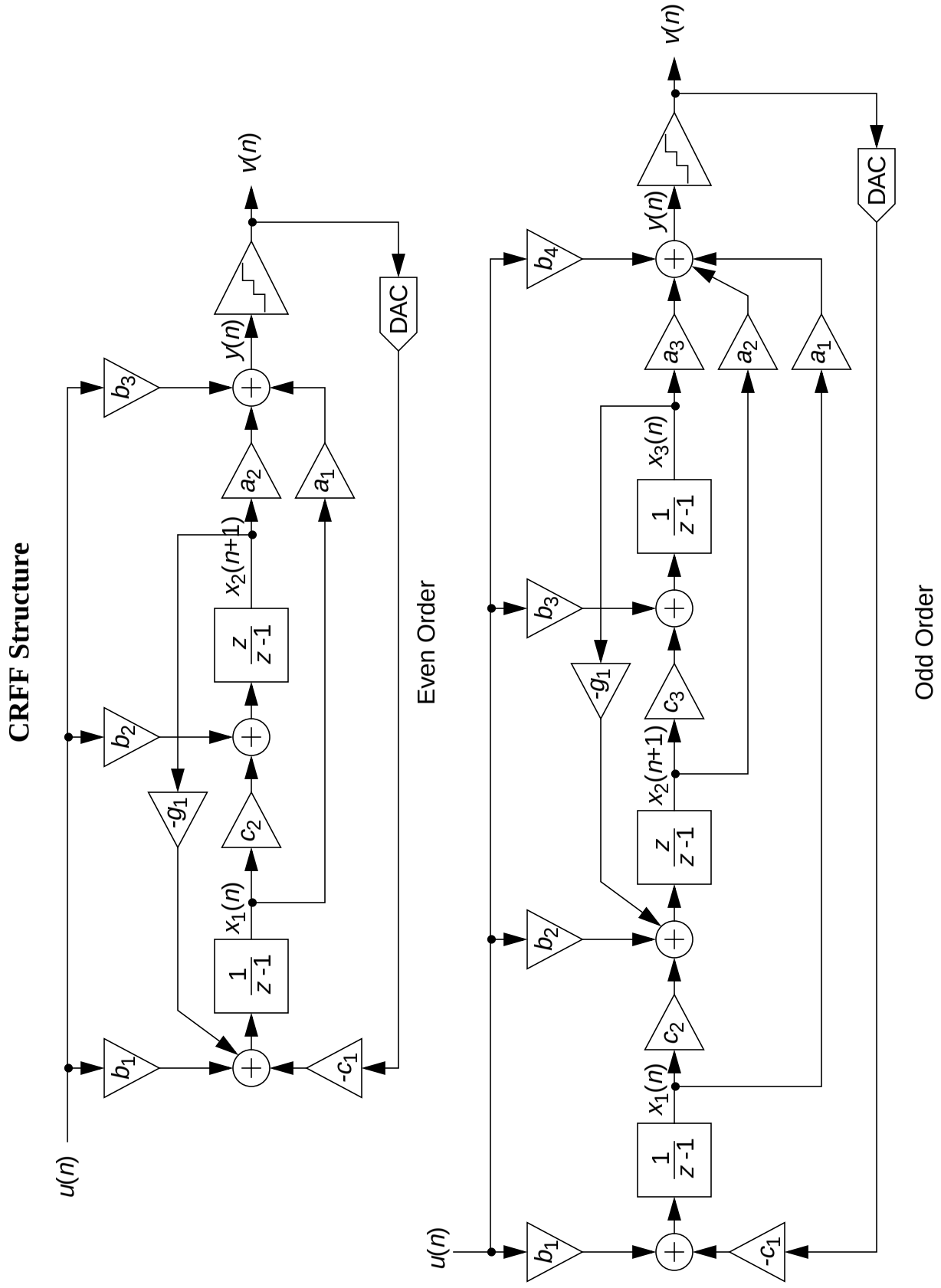
CIFB	Cascade-of-integrators, feedback form.
CIFF	Cascade-of-integrators, feedforward form.
CRFB	Cascade-of-resonators, feedback form.
CRFF	Cascade-of-resonators, feedforward form.
CRFBD	Cascade-of-resonators, feedback form, delaying quantizer.
CRFFD	Cascade-of-resonators, feedforward form, delaying quantizer

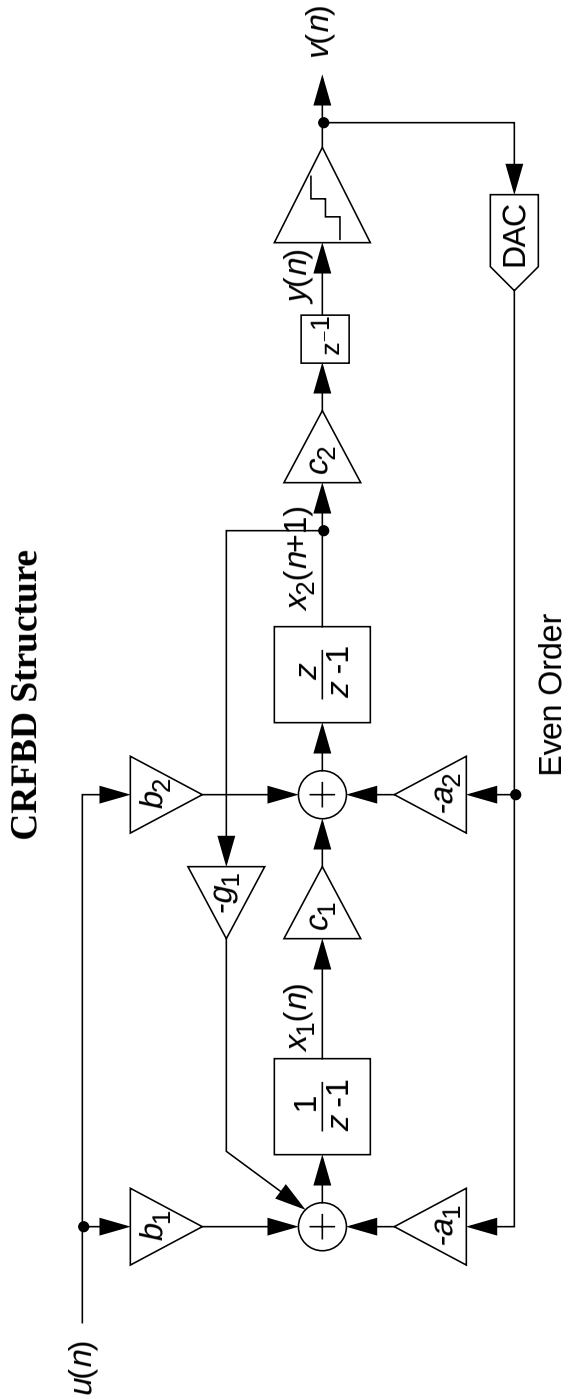
Multi-input and multi-quantizer systems can also be described with an ABCD matrix and Eq. (2) will still apply. For an n_i-input, n_o-output modulator, the dimensions of the sub-matrices are A: $n\times n$, B: $n\times(n_i+n_o)$, C: $n_o\times n$ and D: $n_o\times(n_i+n_o)$.


CIFB Structure
u(n)
v(n)
DAC
Even Order
Odd Order
b1
b2
b3
b4
c1
c2
c3
-g1
-a1
-a2
-a3
x1(n)
x2(n)
x3(n)
y(n)
1
z -1

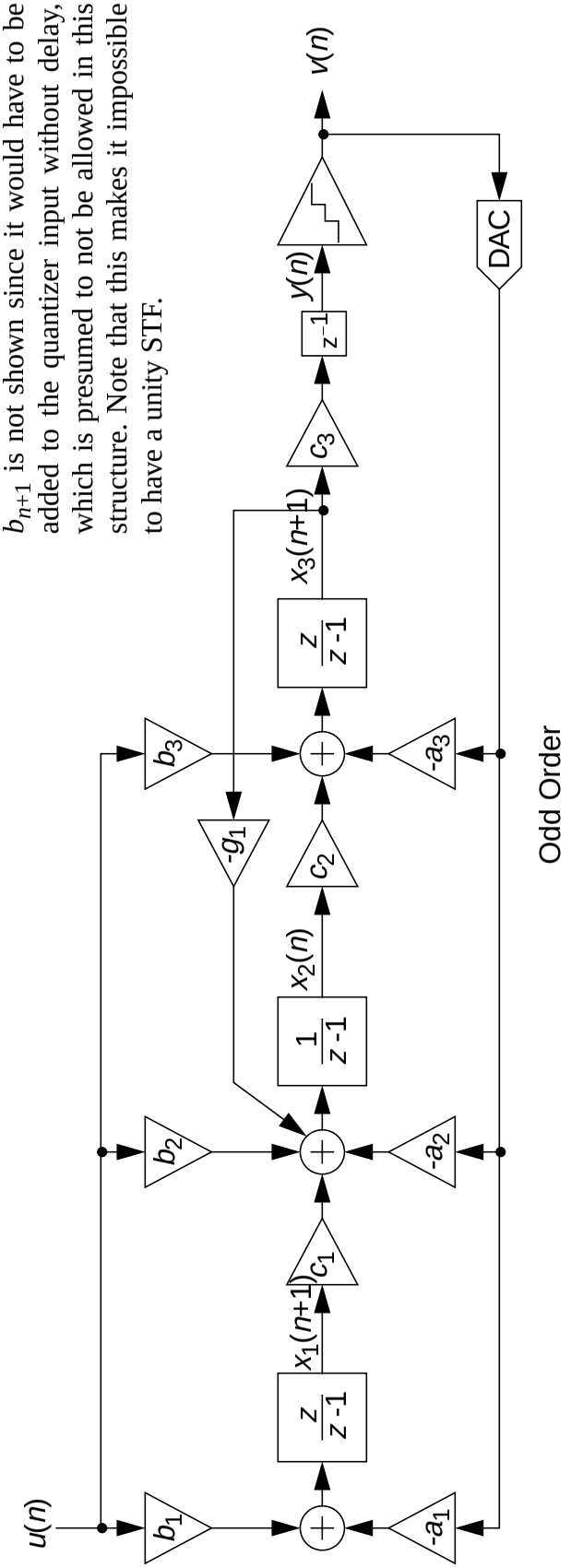

CIFF Structure
u(n)
b1
b2
b3
-g1
c2
-c1
x1(n)
x2(n)
1/(z-1)
1/(z-1)
a2
a1
y(n)
v(n)
DAC
Even Order
u(n)
b1
b2
b3
b4
-g1
c2
c3
-c1
x1(n)
x2(n)
x3(n)
1/(z-1)
1/(z-1)
1/(z-1)
a3
a2
a1
y(n)
v(n)
DAC
Odd Order

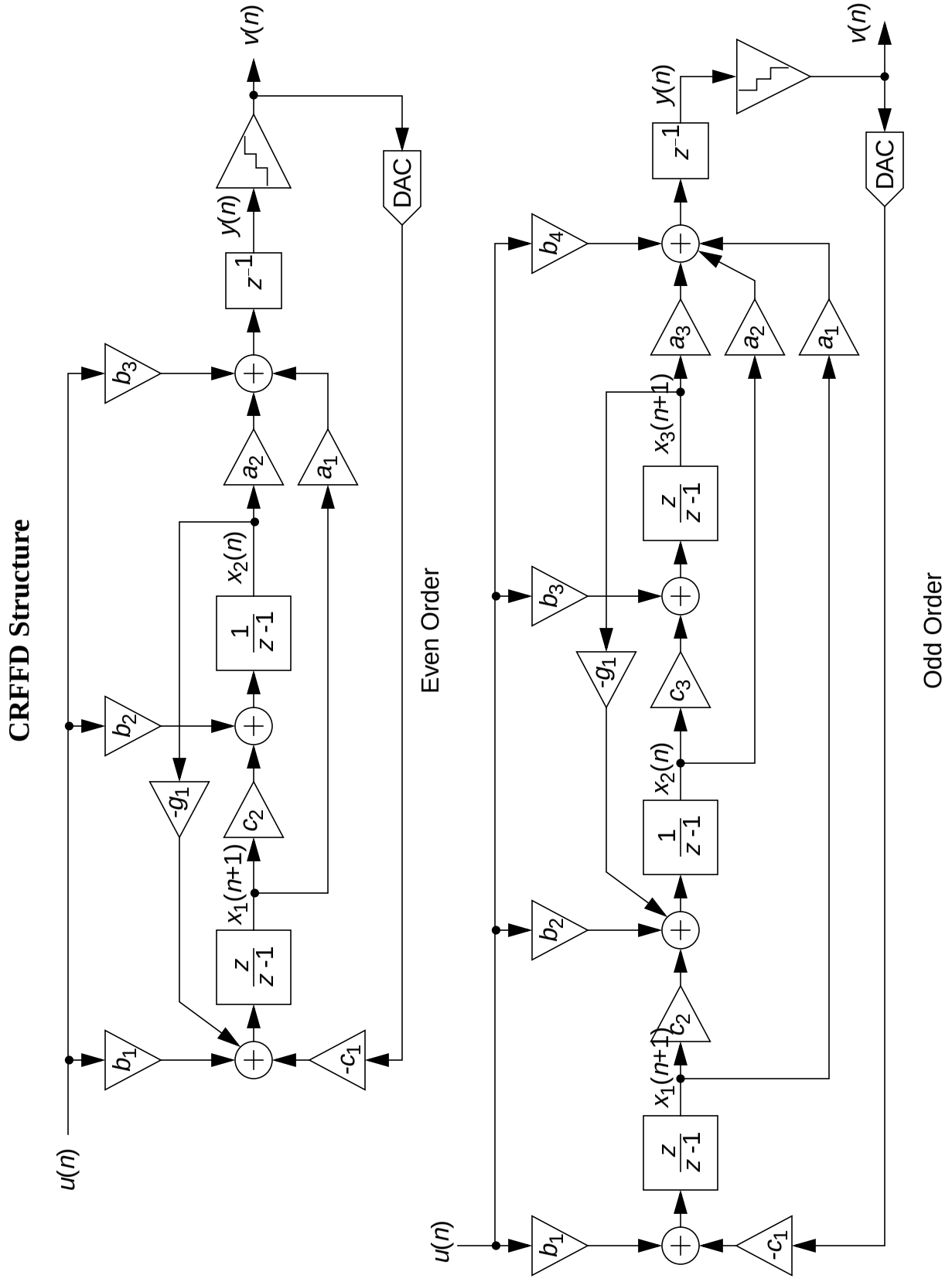

CRFB Structure
u(n)
b1
b2
b3
-g1
c1
c2
-a1
-a2
z / z-1
x1(n+1)
1 / z-1
x2(n)
y(n)
v(n)
DAC
Even Order
Note that with NTFs designed using synthesizeNTF, omission of the b2... coefficients in the CRFB structure will yield a maximally-flat STF.
u(n)
b1
b2
b3
b4
-g1
c1
c2
c3
-a1
-a2
-a3
1 / z-1
z / z-1
x1(n)
x2(n+1)
x3(n)
y(n)
v(n)
DAC
Odd Order


CRFF Structure
u(n)
b1
b2
b3
-g1
c2
-c1
x1(n)
x2(n+1)
1/(z-1)
z/(z-1)
a2
a1
y(n)
v(n)
DAC
Even Order
b4
a3
x3(n)
c3
Odd Order



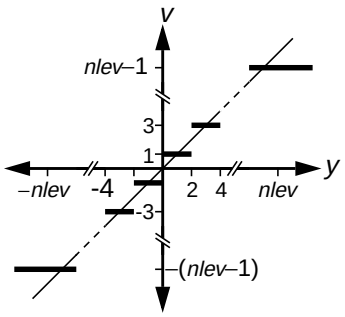
b_{n+1} is not shown since it would have to be added to the quantizer input without delay, which is presumed to not be allowed in this structure. Note that this makes it impossible to have a unity STF.



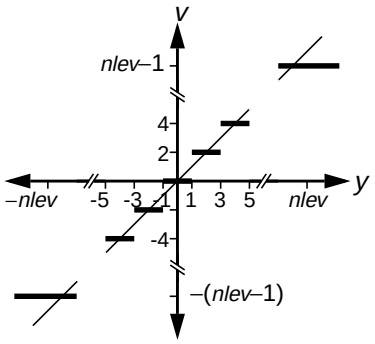

CRFFD Structure
u(n)
b1
b2
b3
-g1
c2
-c1
a2
a1
z/(z-1)
1/(z-1)
x1(n+1)
x2(n)
z-1
y(n)
v(n)
DAC
Even Order
u(n)
b1
b2
b3
b4
-g1
c2
c3
-c1
a3
a2
a1
z/(z-1)
1/(z-1)
x1(n+1)
x2(n)
x3(n+1)
z-1
y(n)
v(n)
DAC
Odd Order

The Quantizer

The quantizer is ideal, producing integer outputs centered about zero. Quantizers with an even number of levels are of the mid-rise type and produce outputs which are odd integers. Quantizers with an odd number of levels are of the mid-tread type and produce outputs which are even integers.



Transfer curve of a quantizer with an even number of levels.



Transfer curve of a quantizer with an odd number of levels.